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Amit Tandon and John Marshall

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Einstein's Tea Leaves and Pressure Systems in the Atmosphere

Amit Tandon, University of Massachusetts–Dartmouth, Dartmouth, MA
John Marshall, Massachusetts Institute of Technology, Boston, MA

Tea leaves gather in the center of the cup when the tea is stirred. In 1926 Einstein[1] explained the phenomenon in terms of a secondary, rim-to-center circulation caused by the fluid rubbing against the bottom of the cup. This explanation can be connected to air movement in atmospheric pressure systems to explore, for example, why low-pressure systems tend to be stormy and high-pressure systems are fair weather. Here, following Einstein's lead, we revisit the tea leaf phenomenon, make the connection with atmospheric pressure systems, and describe an illustrative laboratory experiment.

Tea leaves in a cup

The following quote from Einstein's 1926 article explains the phenomenon,

> "Imagine a flat-bottomed cup full of tea. At the bottom there are some tea leaves, which stay there because they are rather heavier than the liquid they have displaced. If the liquid is made to rotate by a spoon, the leaves will soon collect in the center of the bottom of the cup. The explanation of the phenomenon is as follows: the rotation of the liquid causes a centrifugal force to act on it. This in itself would give rise to no change in the flow of the liquid if the latter rotated like a solid body. But in the neighborhood of the walls of the cup, the liquid is restrained by friction, so that the angular velocity with which it rotates is less there than in other places nearer the center. In particular, the angular velocity of rotation, and therefore the centrifugal force, will be smaller near the bottom than higher up. The result of this will be a circular movement of the liquid of the type illustrated in Fig. 1 which goes on increasing until, under the influence of ground friction, it becomes stationary. The tea leaves are swept into the center by the circular movement and act as proof of its existence."

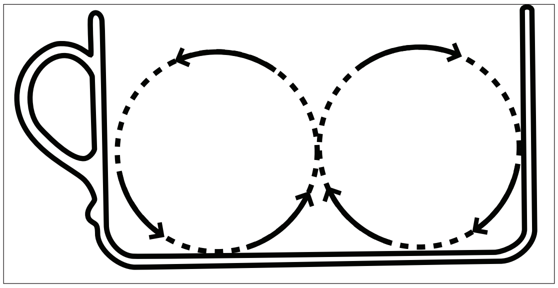

Fig. 1. From Ref. 1. The arrows show the secondary circulation induced by friction between the stirred tea and the bottom of the cup.

The interface region where friction between the fluid and the container takes place is called the *boundary layer*. In the cup, the force arising due to radial pressure difference[2] provides the centripetal force required to keep a fluid parcel moving in a circle, i.e.,

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{V^2}{r}, \qquad (1)$$

where ρ is the density of the fluid, p is the pressure, V is the fluid velocity, and r is the radial distance from the center. Here the left-hand term is called the *pressure gradient force* per unit mass, arising due to the pressure variation in the radial direction. The partial derivative notation simply means that the pressure difference variations are being evaluated at some constant depth. The right-hand term is the *centripetal force* per unit mass. Outside the boundary layer, where the fluid rotates as a solid body, $V = \Omega r$, where Ω is the rate of rotation. Within the boundary layer, however, the fluid velocity decreases, say, from V to v, and the pressure gradient is not balanced due to this lower velocity. This decrease in velocity leads to a net inward acceleration per unit volume of the fluid within the boundary layer, which has the value

$$\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v^2}{r} = \frac{V^2 - v^2}{r}.$$

If $v = V$, there is no net inward force on the fluid and the fluid would simply move in a circle. If $v < V$, then there is a net inward force, and if $v > V$, there is a net outward force. The bottom boundary layer of the teacup corresponds to the former case.

It is insightful for students to view this situation from overhead, not in the usual stationary reference frame, but in a rotating reference frame that has the same angular velocity as the bulk of the fluid in solid body rotation. In this case, it is easier to see the role that the Coriolis force plays in the rotating reference frame. Viewing the cup from the point of view of a reference frame rotating with the bulk of the fluid, the interior fluid is at rest in this frame, and the slower-moving fluid near the bottom appears to move opposite to the rotation of the reference frame velocity Ω with a tangential velocity v_t. (Note, $v_t = 0$ outside the boundary layer and $v_t = -\Omega r$ right at the bottom, so $v_t \leq 0$ in the boundary layer.) We can write the boundary layer velocity v in the frame of reference of the teacup (i.e., the resting frame) as,

$$v = V + v_t. \qquad (2)$$

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Substituting Eq. (2) in the expression for net inward acceleration,

$$\frac{V^2-v^2}{r},$$

we can rewrite it in terms of v_t and Ω as follows:

$$\begin{aligned}\frac{V^2-v^2}{r} &= -\frac{2Vv_t}{r} - \frac{v_t^{\,2}}{r}\\ &= -2\Omega v_t - \frac{v_t^{\,2}}{r},\end{aligned} \tag{3}$$

where it has been assumed that $V = \Omega r$ outside the boundary layer. Here, the first term on the right-hand side represents the Coriolis force, and the second term represents the centrifugal force (both per unit mass) in the rotating frame.[3] The above expression tells us that the net force is inward, since $V > v$. Formally, too, we see the Coriolis force ($-2\Omega v_t$) is inward, irrespective of whether the fluid is stirred clockwise or counterclockwise, since v_t changes sign when Ω is reversed.[4] Thus, in the rotating reference frame of the tea leaves phenomenon, the Coriolis force is always directed inward and is greater than the centrifugal term: the fluid within the boundary layer moves toward the center and takes the tea leaves along with it. This net force leads to a secondary circulation as the fluid moves inward. This secondary circulation must go somewhere, which then demands that the fluid rise near the center and descend near the walls of the cup, just as sketched by Einstein in Fig. 1. Take, for example, a teacup of radius 5 cm, set in motion at $\Omega = 30$ rpm (or $\pi/2$ rad/s) by a stirring spoon. In the rotating reference frame, at the bottom of the cup, the centrifugal term is $5\pi^2$ cm/s^2 (0.493 m/s^2) and directed outward, whereas the Coriolis term is $10\pi^2$ cm/s^2 (0.986 m/s^2) and directed inward. The nondimensional ratio of centrifugal (or, more generally, inertial) term to the Coriolis term is called the Rossby number (Ro), after the famous meteorologist C. G. Rossby. The Rossby number, $v/(2\Omega L)$, can easily be estimated for flows with characteristic length scale L, velocity v, and rate of rotation Ω. For the tea leaves example, Ro is ½, whereas for large-scale atmospheric flows, the Rossby number is typically quite small, closer to 0.1. This number is small for flows that are dominated by rotation. Low Rossby number flows have a number of interesting properties.

Applicability to pressure systems

The secondary circulation discussed above has an important analog in atmospheric pressure systems, the circulation of air around them, and the associated vertical motion within them. One difference from the teacup, of course, is that Ω must now be interpreted as the rotation rate of the Earth, rather than the angular swirl of the tea in the cup: thus Ω is constant, does not change sign, and is positive. Another is that the radius of curvature for pressure systems can be fairly large, reducing centrifugal effects and leading to a small Rossby number. While centrifugal terms in the rotating frame of the Earth are particularly important for small-scale systems such as tornadoes, they are much less important for large-pressure/storm systems. For atmospheric systems, the Coriolis acceleration term is written as $f \times v_t$, where the Coriolis parameter f takes into account the component of Earth's rotation in local vertical direction (opposing gravity), and v_t is the velocity with respect to the rotating Earth. For example, at 40 N, $f = 2\,\Omega_{\text{Earth}} \sin(40) = 6.8 \times 10^{-5}\text{s}^{-1}$. For a tornado of radius 0.5 km and wind speed 20 m/s, the Rossby number is 571, the centrifugal term dominates, and the Coriolis term can be ignored. In contrast, for a pressure system idealized as a circular system of $r = 1000$ km, for 5 m/s wind, the Rossby number is small, 0.07, and the Coriolis term dominates.

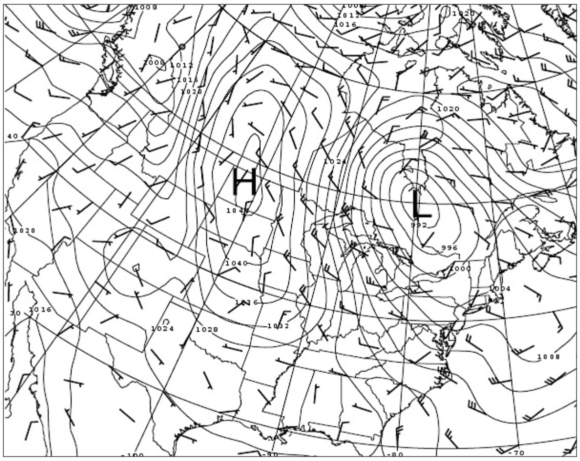

Fig. 2. From Ref. 6. Surface pressure field and surface wind at 12 GMT on Feb. 10, 2008. The contour interval is 4 mbar. High- and low-pressure systems are marked. The line segments represent "wind arrows" that indicate wind direction. By meteorological convention these are drawn without arrowheads; the barbs at the tails of the arrows indicate the wind speed. One long barb represents a wind speed of 5 m/s blowing in the direction of the arrow shaft, away from the barb. One full quiver represents a wind of 5 m/s. We see air circling in a generally counterclockwise direction around the low but spiraling inward, and air circling in a generally clockwise direction around the high but spiraling outward.

While discussing the pressure systems below, we shall assume a small Rossby number, and hence ignore the centrifugal term. In a low-pressure system, the air outside the bottom boundary layer is in a balance where, in the rotating frame, the dominant outward Coriolis force balances the inward radial pressure gradient. This is called the *geostrophic balance*. The air rotates cyclonically (in the same sense as the local vertical component of the rotation of the Earth) for this balance to be achieved. For example, in the Northern Hemisphere the air would circulate in a counterclockwise sense outside the bottom boundary layer, parallel to the isobars. As the ground is approached from above, and the influence of friction becomes important, this balance changes. The friction slows down the primary circular motion, and thus the Coriolis force is lower and no longer sufficient for the fluid to flow paral-

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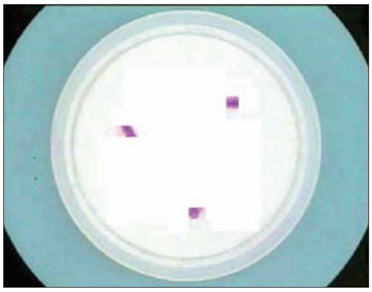

Fig. 3. The rotating platform showing initial distribution of potassium permanganate crystals to illustrate secondary circulation.

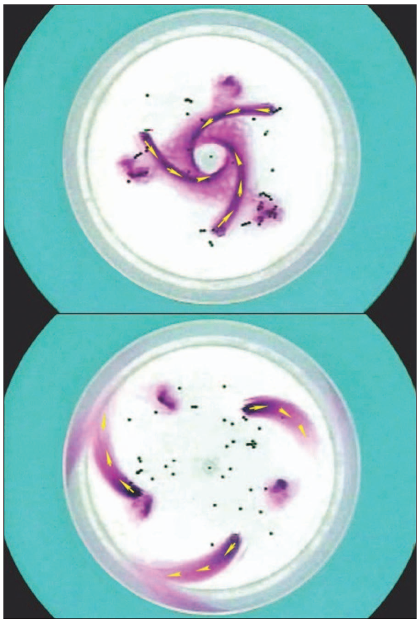

Fig. 4. (at right) The top panel shows near bottom inward circulation, analogus to low-pressure near surface flow, while the bottom panel shows outward circulation near the bottom analogous to the near surface flow for a high-pressure system. (From Ref. 6.)

lel to the isobars. There is a net inward force, and the air thus moves toward the center of the Low, marked L in Fig. 2, much like the tea leaves example. The converse case is also very interesting. Around a high-pressure system, the air rotation is anticyclonic to achieve balance. In this case for the air outside the bottom boundary layer, the inward Coriolis force balances the outward radial pressure gradient.

As the ground is approached, the friction slows down the primary motion (clockwise in the Northern Hemisphere), which reduces the dominant Coriolis force, while the pressure gradient remains the same and thus there is a net outward force in the bottom boundary layer. The air flow acquires a net radially outward component, as seen in Fig. 2, for arrows near the High, marked H.

Demonstration using a rotating platform

Besides tea leaves stirred in a cup, other classroom demonstrations can show the secondary circulation that arises due to the presence of friction. We discuss below a simple laboratory experiment that demonstrates these ideas in a more controlled setting and can be readily applied to both atmospheric low- and high-pressure systems. All one needs is a rotating turntable, a cylindrical container (a large transparent beaker or a cylindrical insert inside a square container works fine), and some potassium permanganate crystals.[5] When rotated at a constant rate, all the water comes into solid body rotation, and so there are no Coriolis or centrifugal accelerations acting. The key experimental requirement is to be able to speed up or slow down (by 10% or so) the rate of rotation of the turntable so as to induce relative motion between the water and the tank, thus creating a frictional boundary layer. The rotating platform can be used in a whole series of experiments to demonstrate atmospheric and oceanic phenomena, as presented in Marshall and Plumb[6] and the “Weather in a Tank” website.[5]

A tank of water is placed on the rotating platform long enough for water to reach solid body rotation. The rotation is cyclonic (counterclockwise), looking down from the top, to mimic the sense of rotation of the Earth. The precise rotation rate is not of great importance, but between 10 and 15 rpm works well. After solid body rotation has been achieved (10 minutes or so, depending on the size of the container), three small applications of potassium permanganate crystals are made at the corners of a equilateral triangle, as can be seen in Fig. 3. If solid body rotation has indeed been achieved, the crystals should fall vertically (note this is only true at rotation rates that are low enough that the free surface does not become significantly concave, depressed in the middle, and rising up to the outside) and settle on the bottom and remain together as three small clouds rather than disperse. We also drop a few colored paper dots on the surface to see the flow outside the boundary layer. As the table is slowed down by a few rpm (about 10%), the permanganate on the bottom traces the near bottom circulation, which is cyclonic and inward, just like a low-pressure system. The paper dots floating on the surface do not go inward. Why does this happen?

The water outside the boundary layer is still rotating with the original fast rotation rate, while the water at the bottom is rotating slower, at the new slower rotation rate. Because of the inertia of the turning fluid, it continues to spin at its original speed and so moves relative to the tank: permanganate streaks are pulled around, not in circles as one might initially expect, but rather inward turning, counterclockwise spirals, as can be seen in the top panel of Fig. 4. A beautiful symmetric pattern is remarkably easy to achieve. This is analogous to the near-surface flow in a low-pressure system, as can be seen by comparing with Fig. 2 (see low-pressure system). The flow at the upper surface is visualized by a few floating paper dots (black

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dots are the most visible in this application). We observe circular, rather than spiraling, motion.

To create an analogy of a high-pressure system, we now simply increase the speed of the turntable by 10% or so (back up to, roughly, its original speed). We observe the dye streaks on the bottom reversing and, over time, spiraling clockwise and outward, as can be seen in the lower panel of Fig. 4. This outward anticyclonic flow is analogous to the surface boundary layer of a high-pressure system.

This secondary flow in the boundary layer has important implications for movement in the vertical direction. The inward flow associated with a low-pressure system leads to rising air near the center of the Low. As this air rises, it expands (pressure always decreases going upward in the atmosphere) and cools.[7] Since the saturation of the air is very strongly dependent on the temperature, as the air cools it may get saturated, and the water vapor may condense out to form—clouds! This is why the low-pressure systems are the ones associated with stormy weather and precipitation. Conversely, high-pressure systems are associated with outward motion in the boundary layer, and hence subsidence. As the air descends, it gets compressed due to the pressure increase, warms, and becomes less and less saturated. Thus, the high- pressure systems are fair-weather systems.

Acknowledgments

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2. Evidence for this pressure difference can be seen by observing the tilt of the free surface of the fluid that rides up the side of the container, resulting in high pressure in the periphery, low pressure toward the center.
3. In the fixed reference frame, the sum of all external forces provides the centripetal and Coriolis acceleration, while in a rotating reference frame, the sum of external forces on the object are balanced by the centrifugal and the Coriolis force. See the James Bond cartoon at xkcd.com/123/.
4. Since $v_t = -\Omega r$, then $-2\Omega v_t = 2\Omega^2 r$ and so is always positive, corresponding to an inward acceleration.
5. “Weather in a tank” website and project are at paoc.mit.edu/labguide/ and paoc.mit.edu/labguide/ekman.html. Readers and students can generate their own maps (similar to Fig. 2) for any day by going to www.paoc.mit.edu/labguide/ekman_atmos.html, clicking on the maps, and following subsequent directions.
6. John Marshall and Alan Plumb, *Atmosphere, Ocean and Climate Dynamics, An Introductory Text* (Academic Press, 2008), p. 131.
7. The overwhelming influence of gravity in the vertical direction implies that the pressure variation is much larger in the vertical than it is in the horizontal.

Amit Tandon *is an associate professor of physics and oceanography at University of Massachusetts Dartmouth. He teaches undergraduate and graduate courses in physics, fluid mechanics and oceanography, and general education courses on weather and climate change related topics. His research interests involve mixing processes in the upper ocean, and use of process studies to quantify the role of various sub-grid scale phenomenon for predictive ocean circulation models or long-term climate studies.*
atandon@umassd.edu

John Marshall *is a professor in the Department of Earth, Atmospheric, and Planetary Sciences at MIT. His research has focused on important, and usually difficult, problems of the ocean circulation involving interactions between motions on different scales, using theory, laboratory experiments, and new innovative approaches to global ocean modeling pioneered by his group at MIT.*
Both authors strongly prefer the use of hands-on experiments in their teaching.

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