Thermal Equilibration of Planetary Waves

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ABSTRACT

Equilibration of planetary waves toward free-mode forms, steady solutions of the unforced, undamped equations of motion, is studied in a three-level quasi-geostrophic model on the hemisphere. A thermal mechanism is invoked, parameterized as a Newtonian process $Q = -\gamma(T - T^*)$, relaxing the atmosphere toward a radiative–convective equilibrium temperature T^* on γ^{-1} time scales. If T^* is chosen to project onto the class of finite-amplitude stationary Rossby waves, T can closely approach T^* if, simultaneously, the surface winds vanish switching off the Ekman layers at the surface. The equilibrated state is characterized by vertical phase lines, zero surface winds, vanishing diabatic heating rates and a temperature field that is phase-locked with T^* corresponding to ridges over the oceans and troughs over the land. The form of the equilibrated planetary wave is contrasted with the classical thermally forced response obtained when T^* does not have free-mode form. Anomaly fields calculated from the model, the difference between equilibrated and nonequilibrated waves, have a characteristic pattern which is reminiscent of Rossby wave trains.

1. Introduction

Since the classical studies of orographic (Charney and Eliassen 1949) and thermal (Smagorinsky 1953) forcing of planetary waves there has been a continuing debate concerning the relative importance of diabatic and orographic forcing in determining the extra-tropical wave pattern—for recent reviews see Hoskins and Karoly (1981) or Held (1983). In such studies a seasonal or climatological diabatic forcing, Q, is considered to be a fixed function of space (for example, as deduced from a residual budget calculation) and one seeks the linearized response about a zonal flow subject to lower boundary conditions set by orography.

In reality of course Q is not a fixed function of space but is strongly controlled by the amplitude and phase of the planetary waves themselves. For example, the distribution of diabatic heating in middle latitudes is strongly modulated by the position of the storm tracks, being largely associated with latent heat release. If the oceanic storm track is interrupted by a high amplitude ridge associated with a blocked flow downstream, one might expect diabatic heating rates over the storm track to be anomalously low. Thus Q should not be regarded as a fixed function of space and time. Indeed Shutts (1987) has suggested that if one keeps Q fixed then the magnitude of the thermal response may be underestimated because, as described below, the possibility of thermal equilibration is excluded.

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One such model in which the heating is flow dependent is that of Doos (1962) in which Q is parameterized as a Newtonian process $Q = -\gamma(T - T^*)$. As discussed in Shutts (1987), T^* can be thought of as a radiative–convective temperature appropriate to a particular geographical region, and γ^{-1} a time constant dependent on the nature of the physical processes responsible for adjustment. T^* and γ^{-1} are likely to be strongly influenced by the nature of the underlying surface and particularly the contrast between land and sea. For example in winter the vertical temperature profile over the ocean can rapidly adjust to an equilibrium temperature strongly influenced by the sea-surface temperature (deep convection over a warm ocean can bring the troposphere close to the saturated adiabat corresponding to the SST). Over land in winter, where the equilibrium temperature is uniformly low, the adjustment time scale can be very much longer and set primarily by radiative processes. In winter conditions, superimposed on its large-scale meridional gradient, T^* can be expected to be low over the continents (North America and Euroasia), high over the oceans, and so exhibit a wavy form.

Consider, for example, the case in which the spatial form of T^* happens to be compatible with a solution of the unforced/undamped equations of motion (a stationary free Rossby wave for example). The possibility then exists that T can closely approach T^* with markedly reduced diabatic heating rates. This process may be called thermal equilibration following Shutts (1987). The dangers of attributing the thermal response to a fixed forcing are now apparent for the diabatic heating computed as a residual in the thermodynamic equation may be vanishingly small on the scale of the stationary Rossby wave and therefore dominated by noise. Consequently there ought to be little confidence in the linear solution to fixed forcing when the flow is near equilibration. The classical picture of thermal forcing places emphasis on the forced response; instead, the equilibration viewpoint interprets the Q field as a measure of the degree to which free-mode form has been attained.

A parallel difficulty arises in theories of orographic forcing if the lower boundary condition is linearized to represent the orographic uplift on a zonal flow impinging on the mountain. As discussed by Saltzman and Irsch (1972), orographic effects also force flow around obstacles; in the limiting case where streamlines and mountain height contours are coincident, the induced vertical velocity is zero. Appropriate nonlinear boundary conditions which are capable of representing this limit have been recently studied by Chen and Trenberth (1987). The analogous diabatic process, that of thermal equilibration, can occur in thermally forced problems if Q is allowed to be a function of the flow. Such a representation of the diabatic heating field, used in conjunction with a more complete kinematic lower boundary condition, would reduce the importance of orographic relative to thermal effects compared to that implied by the classical linear forced model.

Here, using a three-level quasi-geostrophic hemispheric model at truncation T15, we study thermal equilibration in isolation. A development of the β-plane channel study of Mitchell and Derome (1983) to a sphere, the mechanism considered by Shutts (1987) is illustrated with a fully nonlinear model which has many degrees of freedom. In the present study we adopt, as prototype free modes, the class of stationary Rossby waves which form a discrete set determined by model geometry and the basic state flow field. Such waves rely for their existence on significant reflection of wave activity from the tropics, an assumption that will be tested numerically in our model.

In section 2 the process of equilibration is introduced in the context of a linear model of thermal forcing in which the perturbation streamfunction about a solid body rotation is sought in response to a diabatic heat source represented as a Newtonian process $Q = -\gamma(T - T^*)$. The spectrum of thermally forced response is illustrated: if T^* has free mode form, then equilibration is possible resulting in a large-amplitude response in phase with T^* and vanishing diabatic heating rates; the nonequilibrated response is of smaller amplitude, out of phase with T^*, and associated with larger diabatic heating rates.

In section 3 the form of possible equilibrated states is studied by solving analytically for finite amplitude quasi-geostrophic free modes on a hemisphere. Rossby wave solutions can be readily found if the axi-symmetric component of the flow is such that the function $\Lambda = d\bar{q}/d\bar{\psi}$, where $\bar{q}$ is the potential vorticity and $\bar{\psi}$ is the streamfunction, only depends on height. Scatter diagrams of q against ψ computed from Northern Hemisphere ECMWF fields are presented which indeed suggest that the extratropical flow is close to free-mode form with linear (q, ψ) relationships and therefore expandable in terms of the class of stationary, free Rossby waves.

In section 4 equilibration toward these solutions is studied in a three-level quasi-geostrophic hemispheric model forced by a diabatic heating $Q = -\gamma(T - T^*)$ and spun down by Ekman friction at the ground. These experiments are highly idealized with no attempt to simulate observed flows. Our purpose here is to show that if T^* is chosen to have free-mode form then equilibration toward T^* readily occurs provided that the vertical structure of the wave is consistent with zero pressure perturbation at the surface. The thermally equilibrated wave, characterized by vertical phase lines, zero surface winds and vanishingly small diabatic heating rates, is contrasted with the more familiar nonequilibrated response. Interestingly the anomaly fields, the difference between equilibrated and nonequilibrated responses, have a characteristic pattern which has the appearance of Rossby wave trains. Finally, equilibration experiments in the presence of a band of tropical easterlies of uniform potential vorticity are carried out and it is shown that their presence need not lead to a significant reduction of wave amplitudes in the middle latitudes of the model.

In section 5 implications of the study are discussed for our understanding of the Northern Hemisphere wintertime long-wave pattern, its anomalies and prediction. In particular we speculate on the possibility that equilibration toward free-mode states may be a controlling influence in the evolution of the atmosphere.

2. Linear theory

The thermal equilibration mechanism is introduced in the context of a three-level quasi-geostrophic spherical, hemispheric model. The flow is linearized about a zonal flow in solid body rotation which varies in the vertical. The model is forced by an entropy source term Q, where Q is parameterized as a Newtonian process. The linear model will be used to interpret the numerical results of section 4.

Standard quasi-geostrophic equations are used, expressed in spherical coordinates for a dry atmosphere using $h = \ln(p_0/p)$ as the height coordinate where p_0 is the constant average surface pressure. The nondimensional potential vorticity equation is

$$\frac{\partial q}{\partial t} + J(\psi, q) = \frac{G_0}{p_*} \frac{\partial}{\partial h}\left(\frac{p_* Q}{S}\right) \tag{1}$$

where the time and length scales are the inverse of the rotation frequency Ω^{-1} and the radius of the earth a, respectively,

$$q = \nabla^2\psi + 2\sin\theta + \frac{G_0}{p_*}\frac{\partial}{\partial h}\left(\frac{p_*}{S}\frac{\partial\psi}{\partial h}\right)$$

is the potential vorticity and ψ is the streamfunction,

$$p_* = p/p_0$$

$$G_0 = \sin\pi/4$$

$$S = R\left(\frac{\partial T_0}{\partial h} + \kappa T_0\right)\Big/(f_0\Omega a^2), \quad \text{the static stability}$$

$$Q = (\text{rate of heating in K s}^{-1}) \times R/(f_0\Omega^2 a^2)$$

and f_0, T, R, κ have their usual meanings. It should be noted that, as is required for energetic consistency, we assume here that the Coriolis parameter is constant in the stretching contribution to the potential vorticity.

Stationary perturbations are sought to an atmosphere rotating such that the angular velocity of rotation is a function of height alone:

$$\psi = \bar{\psi} + \psi', \quad \bar{\psi} = -\chi(h)\sin\theta \qquad (2)$$

implying a zonal flow

$$\bar{u}(\theta, h) = -\cos\theta\frac{\partial\bar{\psi}}{\partial\mu} = \chi(h)\cos\theta$$

where θ is the latitude and $\mu = \sin\theta$. Substituting the expression for the streamfunction Eq. (2) into Eq. (1) and neglecting the products of primed quantities, $J(\psi', q')$,[1] yields

$$J(\bar{\psi}, q') + J(\psi', \bar{q}) = \chi\left(\frac{\partial q'}{\partial\lambda} - \frac{\partial\psi'}{\partial\lambda}\frac{\partial\bar{q}}{\partial\bar{\psi}}\right)$$
$$= \frac{G_0}{p_*}\frac{\partial}{\partial h}\left(\frac{p_* Q}{S}\right) \qquad (3)$$

where λ is the longitude. If ψ' and Q are chosen to be of the form

$$\psi' = \mathrm{Re}\{F(h)\bar{P}_n^m(\mu)e^{im\lambda}\} \qquad (4a)$$

$$Q = \mathrm{Re}\{Q_*(h)\bar{P}_n^m(\mu)e^{im\lambda}\} \qquad (4b)$$

where $\bar{P}_n^m(\mu)$ is the normalized associated Legendre function, then the complex function for the vertical structure $F(h)$ is given by

$$\frac{G_0}{p_*}\frac{\partial}{\partial h}\left(\frac{p_*}{S}\frac{\partial F}{\partial h}\right) - [\Lambda(h) + n(n+1)]F$$
$$= \frac{G_0}{i\chi m p_*}\frac{\partial}{\partial h}\left(\frac{p_* Q_*}{S}\right) \qquad (5a)$$

[1] It should be mentioned that certain-finite amplitude-forced solutions of Eq. 3 can be found in which $J(\psi', q')$ vanishes identically (see Derome 1984) provided that the forcing is monochromatic and of appropriate scale.

where

$$\Lambda(h) = \frac{d\bar{q}}{d\bar{\psi}} = -\frac{\cos\theta}{\bar{u}}\frac{d\bar{q}}{d\mu} \qquad (5b)$$

is set by the form of zonal flow.

As discussed in detail by Butchart et al. (1989), Eq. (5) has the form of a Schrodinger equation in which the role of the "potential function" is played by Λ. It relates the vertical structure of the planetary wave excited by the source on the rhs to its horizontal structure. Another persective from which to view Eq. (5) is provided by the classical studies of baroclinic instability of Eady (1949) and Green (1960). In the case when $Q_* = 0$, it is closely related to the eigenfunction equation for neutral modes of the baroclinic instability problem posed by $\bar{u}(z)$.

For the form of zonal flow about which we have chosen to linearize [Eq. (2)] Λ only depends on χ and thus is a function solely of height, and so separable solutions can be found. Shutts (1978) obtains solutions to the continuously stratified problem [Eq. (5)] for simple choices of $\chi(h)$ with Q a prescribed and fixed function of space. Here, however, Q is represented as a Newtonian heating law of the form

$$Q = -\gamma(T - T^*) \times R/(f_0\Omega^2 a^2) \qquad (6)$$

where $T^* = A(h)\bar{P}_n^m e^{im\lambda}$ is the equilibrium temperature field, to be specified,

$$T = \frac{\partial\psi}{\partial h}a^2\Omega f_0/R$$

in Kelvin, and γ^{-1} is a time constant in seconds. Conventionally, Q is specified a priori whereas in this formulation only T^* is specified. Consequently, the heating is no longer independent of the flow but is a strong function of it.

Rather than considering the continuously stratified problem, a vertical discretization of Eq. (5) is chosen which is identical to the three-level quasi-geostrophic numerical model used in section 4 and described in the Appendix. The vertical structure of the wave response $F(h)$ is found for a given $\Lambda(h)$ and T^* by solving Eqs. (5) and (6). The boundary condition applied at the top of the model is $\omega = 0$ which is only appropriate for waves trapped in the troposphere. For ultralong waves in winter conditions in which the n of the horizontal $\bar{P}_n^m$ structure is less than 4, this boundary condition is not appropriate since such waves are not trapped and energy is propagated into the stratosphere (for example, see Shutts 1978). Therefore, to be consistent with our chosen top boundary condition, we will only consider those waves with $n \geq 4$. The bottom boundary condition invokes an Ekman layer $\omega = \epsilon\nabla^2\psi_s$, where ψ_s is the extrapolated streamfunction at the surface. There is no topography.

Our main objective is to illustrate how the wave response varies as the meridional structure of T^*, set by

n, changes (or equivalently, for fixed T^*, as the zonal wind changes). We have chosen a T^* with $m = 3$ and zero phase tilt in the vertical with amplitudes of 5 K at both thermodynamic levels of the model, a radiative–convective relaxation time scale of $\gamma^{-1} = 10$ days and an Ekman friction spindown time of 3.75 days. The zonal flow is a solid body rotation of strength 4.8, 19.9 and 37.3 m s^{-1} (at the equator) at the three dynamic levels. The corresponding Λ at level 1 to 3 are 227, -105 and -134 in nondimensional units (5.58×10^{-12} m^{-2}, -2.58×10^{-12} m^{-2}, -3.29×10^{-12} m^{-2} in dimensional units), the minimum at the upper level being consistent with the data given by Derome (1984) and the data presented in section 3.

If Λ is constant everywhere, for example when $\bar{u} = \chi \cos\theta$, $q = \alpha \sin\theta$ with χ and α are constant, then thermal forcing can only excite internal modes. Indeed in thermally forced planetary wave theory attention is more often than not focused on planetary-scale baroclinic modes (wavenumber one, two and three) since it is only on the largest scales that the thermal response to fixed forcing is significant [m divides the rhs of Eq. (5)]. Here, however, we emphasize the pseudo-barotropic response (no sign-changes in the vertical) possible when Λ is a function of height. Observations—see section 3—show that Λ is a strong function of height; in this case thermal forcing can and will excite the pseudobarotropic mode.

The solution of Eq. (5) (vertically discretized at three levels) as n varies is given in Table 1. When $n < 6$, i.e. the scale of the wave is larger than that of the stationary free Rossby wave, then the height field at level 1 is upstream of T^*; the wave tilts eastward corresponding to heat transport from pole to equator by the wave. There is evidence of equatorward heat transport in the summer of the Northern Hemisphere (Eliassen 1958) when the ultralong waves (with small n) are trapped by easterlies in the stratosphere. When $n > 6$ we have the more familiar response in which the wave is downstream of T^* and tilts westward corresponding to heat transport from equator to pole. This downstream response will be illustrated with our spectral model in section 4. In the conventional linear model with fixed forcing, the prescribed thermal forcing at the various levels are in phase. This agrees with the forcing $-\gamma(T - T^*)$ shown in Table 1, except for negligible difference at $n = 8$. However, here the forcing $-\gamma(T - T^*)$ is found as part of the solution and is not prescribed.

The eastward and westward tilting regimes are separated by the interesting case of $n = 6$ corresponding to the free stationary Rossby wave. If $n = 6$, $T = T^*$ and the height field is in phase with T^*. The wave is "locked" onto the T^* pattern and the heating rate reduced to zero. For the particular choice of $\bar{u}$ and T^* chosen here, the (linearly extrapolated) surface streamfunction also vanishes corresponding to zero surface wind. The wave amplitude is at its maximum and the phase lines are vertical, implying no heat transport. This special response can be called the *equilibrated* response (because of the switching off of the forcing), or the *resonant* response (because it corresponds to the excitation of the free stationary wave with the largest possible amplitude set by T^*). Attention should be drawn to the following two points in regards to equilibration/resonance. First, both sides of Eq. (5) vanish but finite values are obtained for F set by the amplitude of T^*. This should be contrasted with conventional thermally forced linear models in which the forcing on the rhs of Eq. (5) is fixed; then the amplitude of F is infinite at resonance and therefore indeterminate. The sensitivity of the conventional linear model close to resonance is often removed by invoking dissipative processes. Second, the vanishing of the rhs at T approaches T^* removes the dependence of the response on m and χ, see Eq. (5). This suggests that at equilibration the same equation is applicable to different wavenumbers m and, furthermore, that the restrictive form of the zonal velocity (solid body rotation) adopted in the linear model can be relaxed. It will be shown in the next section that this is indeed the case and, furthermore, that the possible equilibrated states are free modes of the nonlinear quasi-geostrophic potential vorticity equation.

When Ekman bottom friction is not applied, the phase lines are vertical except for discontinuous changes of 180°. The equilibrated response described above, however, remains unaffected since the associated surface winds vanish. However, as will be discussed in section 4, surface friction is vital in allowing the steady solution to be sustained in our numerical integrations by constraining the barotropic component of the response. There are other ways of constraining the barotropic component but our choice of vanishing surface winds (and hence Ekman fluxes) seems broadly consistent with the omission of explicit or parameterized eddy flux divergences in our forcing Q.

TABLE 1. Amplitudes and phases of wave responses to forcing $\gamma(T^* - T)$, $T^* = 5\text{ K} \times \bar{P}_n^3 e^{i3\lambda}$.

n	Level	ψ (dam, °)	$T^* - T$ (K, °)	T (K, °)
4	3	(3.4, 42°)		
	2	(1.4, 31°)	(4.3, 347°)	(1.3, 49°)
	1	(0.8, 261°)	(4.3, 347°)	(1.3, 50°)
5	3	(5.1, 24°)		
	2	(2.6, 17°)	(3.7, 347°)	(1.6, 31°)
	1	(0.6, 298°)	(3.8, 347°)	(1.6, 32°)
6	3	(20, 0°)		
	2	(12, 0°)	(0, 0°)	(5, 0°)
	1	(4, 0°)	(0, 0°)	(5, 0°)
7	3	(7.5, 171°)		
	2	(5.0, 179°)	(6.6, 354°)	(1.7, 156°)
	1	(2.8, 204°)	(6.6, 354°)	(1.7, 154°)
8	3	(3.1, 154°)		
	2	(2.1, 169°)	(5.5, 354°)	(0.7, 126°)
	1	(1.6, 204°)	(5.4, 353°)	(0.8, 121°)

In this section, it has been suggested that the planetary wave response to thermal forcing ranges from the equilibrated regime at resonance with vertical phase lines to westward and eastward tilting regimes on either side of resonance. It is possible that the atmosphere ranges between these regimes, on a radiative–convective equilibrium timescale as adjustments of the zonal wind and the form of T^*, move the response on and off resonance. This aspect is further investigated with our spectral model in section 4.

3. Finite amplitude free modes on a sphere

In the linear model of section 2 we saw that if T^* has a form compatible with a stationary free Rossby wave—i.e., it is a solution of the homogeneous problem [Eq. (5) with the rhs set to zero], then equilibration towards T^* is possible. In linear theory analytical progress is made possible by linearizing about an atmosphere in solid body rotation in which $\bar{u} \propto \cos(\text{lat})$. More generally, however, we are interested in all classes of solution of the unforced/undamped equations of motion, $\mathbf{v} \cdot \nabla \mathbf{q} = 0$. There are two broad categories of such free-modes amenable to analytical study: the class of Rossby waves and the class of modons. Recently modons have been proposed as prototypes for atmospheric blocking (see for example McWilliams 1980; Haines and Marshall 1987; Verkley 1987; Butchart, Haines and Marshall 1989) and are characterized by the function Λ taking on different values on open and closed streamlines at upper levels in the troposphere. Here, however, to illustrate the equilibration mechanism, we will focus attention on the class of free Rossby waves in which Λ is only a function of height. In the construction of such solutions it is unnecessary to neglect any higher-order terms; all that must be assumed is that there be a linear functional relationship between $\bar{q}$ and $\bar{\psi}$ at each level in the atmosphere:—see for example Derome (1984) or White (1986) and section 3b.

a. Stationary free Rossby waves

Solutions are sought to the nonlinear equation

$$J(\psi, q) = 0 \tag{7}$$

of the form

$$\psi = \bar{\psi} + \psi', \quad \psi' = F(h) \sum_{m \neq 0} A_m \bar{P}_n^m e^{im\lambda} \tag{8a}$$

where

$$\bar{\psi} = \bar{q}/\Lambda \tag{8b}$$

is an axisymmetric flow and

$$\psi' = q'/\Lambda \tag{8c}$$

the departure from it. It can be easily seen that the sum of Eq. (8b) and (8c) satisfies Eq. (7) as long as Λ is a function of height only; each wavy component has the same vertical structure and the same n of the normalized associated Legendre function. It follows from this form of ψ' that Eq. (8c) reduces to

$$\frac{G_0}{p_*} \frac{\partial}{\partial h} \left(\frac{p_*}{S} \frac{\partial F}{\partial h} \right) - [\Lambda(h) + n(n+1)]F = 0, \tag{9}$$

which is Eq. (5) with the rhs set to zero.

It can be immediately seen that since F is a function of h only, Λ must also be a function of h only, consistent with our assumption. Solutions can be found by either specifying the integer n and the vertical structure F, thus defining Λ, or specifying Λ and solving the eigenvalue problem in F with n as the eigenvalue. We further note that the vertical structure of the waves should be chosen in such a way that the extrapolated value at the surface is zero since, otherwise, Ekman friction will move the flow off resonance. In the limiting case of F having the form (1, 1, 1), i.e., independent of h, then Λ and $\bar{u}$ are also independent of h and the system is barotropic. However, such barotropic Rossby waves cannot be locally excited because diabatic forcing has no projection onto the barotropic mode. It should be remembered, however, that in the real atmosphere Λ varies with height and the remote response to thermal forcing can be equivalent barotropic, as in Hoskins and Karoly (1981).

Equation (8b) for the zonal flow can be written as

$$\nabla^2 \bar{\psi} + \frac{G_0}{p_*} \frac{\partial}{\partial h} \left(\frac{p_*}{S} \frac{\partial \bar{\psi}}{\partial h} \right) - \Lambda \bar{\psi} = -2\mu. \tag{10}$$

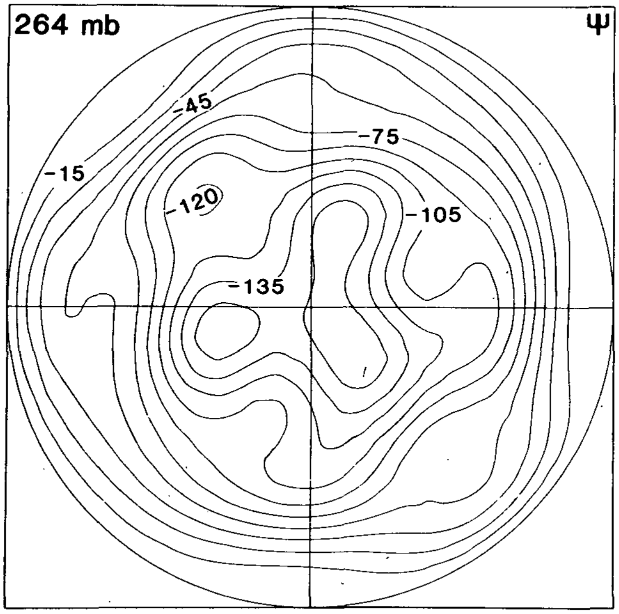
The particular integral is $F_{SB}(h)\mu$ (note $\mu = P_1^0(\mu)$ and $\nabla^2 \mu = -2\mu$). It should be noted that, $|F_{SB}|$, the magnitude of the solid body rotation, increases as n decreases since a strong westerly zonal flow is needed to bring to rest fast-propagating free Rossby waves of very large scale. Since Eq. (10) with the rhs set to zero is identical to Eq. 9, the complementary function for Eq. (10) is $A_0 F(h) \bar{P}_n^0$, where A_0 is an arbitrary constant; thus $\bar{\psi} = F_{SB}\mu + A_0 F \bar{P}_n^0$. In fact, the $\bar{P}_n^0$ component could be incorporated into ψ' by allowing $m = 0$ in the definition Eq. (8a). The zonal flow now consists of the solid body rotation together with the $\bar{P}_n^0$ component whose amplitude can be adjusted arbitrarily. In this way the necessity to adopt the solid-body zonal flow of the linear problem is overcome. Moreover the wavy components ψ' also have arbitrary amplitudes and phases and a rich variety of solutions can be constructed.

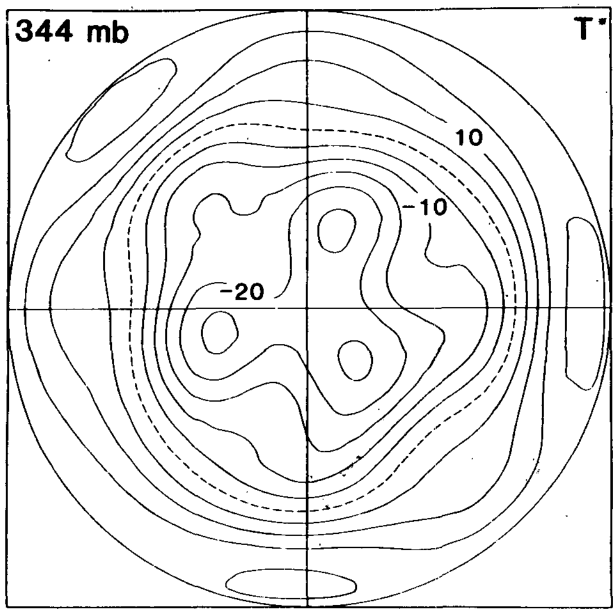
To illustrate the possible complexity of such free-mode configurations, Fig. 1 shows one such solution for a three-level discretization of Eqs. (9) and (10). This planetary wave will be thermally excited in section 4. We choose $n = 8$ and the vertical structure of the waves to be (1, 3, 5). Λ takes the value of 197, −135, and −164 in nondimensional units ($4.77 \times 10^{-12}\ \text{m}^{-2}$, $-3.32 \times 10^{-12}\ \text{m}^{-2}$, $-4.03 \times 10^{-12}\ \text{m}^{-2}$ in dimensional

TABLE 2. Components of a free mode for $n = 8$.

	Amplitude at level 1 to 3 (dam)	Phase in wave space (deg)
$\bar{P}_8^1$	0.85, 2.55, 4.3	166
$\bar{P}_8^3$	3.75, 11.25, 18.75	37
$\bar{P}_8^5$	1.96, 5.89, 9.8	129

units) at levels 1 to 3, broadly consistent with the data (Figs. 2 and 3) presented in the next section. The contribution of the particular integral to the zonal flow is the solid body rotation whose velocities at the equator are 1.5, 10.7, and 22.7 m s^{-1} at levels 1 to 3. For simplicity the amplitude of $\bar{P}_8^0$ has been set to zero and therefore the flow consists of a superposition of solid-body rotation and wavy components only. The wavy components are $\bar{P}_8^1$, $\bar{P}_8^3$, $\bar{P}_8^5$ whose amplitudes and phases are given in Table 2. Figure 1a shows the streamfunction at 264 mb with wavenumbers 3 and 5 dominating at high and middle latitudes respectively, Fig. 1b the temperature field at 344 mb, and Fig. 1c the wavy component of T^*. The troughs of wavenumber 3 are regions of low T^*, which can be thought of as characterizing the continents; the ridges are regions of high T^* characterizing the oceans in winter seasons. The maximum temperature difference between "continents" and "oceans" along a latitude circle, 13.7 K,





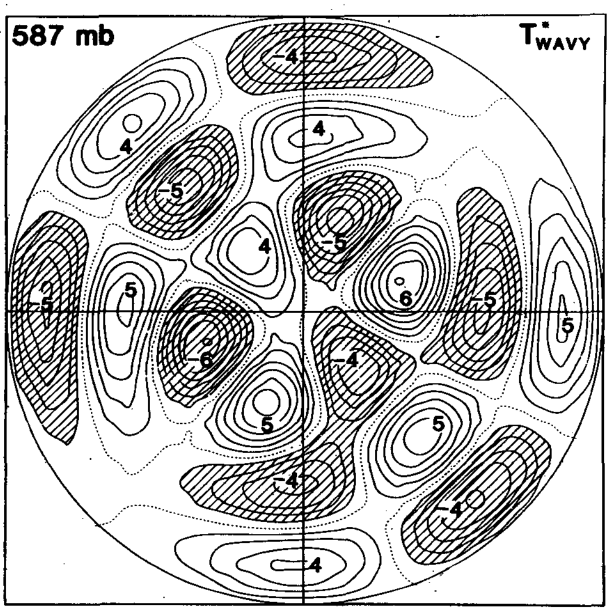


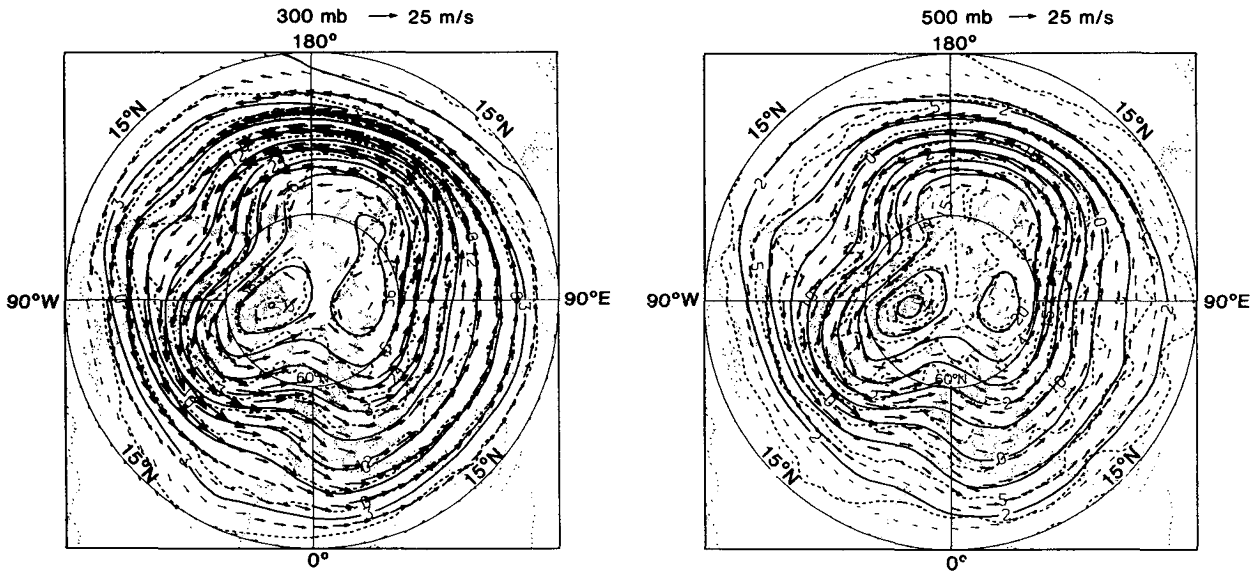
FIG. 1. (a) Streamfunction of the free mode (see Table 2) with $n = 8$ at 264 mb. Contour interval 15 decameters; (b) T^* of the quasi-free mode at 344 mb. Contour interval 5 K; (c) wavy component of T^* at 587 mb. Contour interval 1 K.

is found at 60°N. There are clear regions of strong diffluence and confluence which are not dissimilar to observed flow configurations.

b. Observations of Λ

At first sight it might seem that a zonal flow satisfying Eq. (10), implicitly assuming that $\Lambda = \Lambda(h)$, places undue restrictions on the form of the zonal flow. However, there is observational evidence that the time-mean flow away from surface boundary layers is only a small departure from free-mode form and, moreover, is close to free modes in which the streamfunction is linearly related to the potential vorticity at each level in the atmosphere. Derome (1984) draws attention to this remarkable fact and presents supporting observations from the zonal-average monthly mean flow during January 1979.

In view of the importance of Λ to the theory of finite-amplitude free modes, the form of which is crucial to the ideas being explored in this study, Fig. 2 shows time-mean streamfunction and potential vorticity fields at 300, 500 and 700 mb over the Northern Hemisphere for the period December, January and February of 1986–87. There is a marked tendency for the ψ contours to run parallel to the q contours. Even more remarkable are the functional relationships revealed in Fig. 3 at 300 mb, 500 mb and 700 mb obtained by plotting scatter diagrams of q against ψ at each $5° \times 5°$ grid point over the hemisphere poleward of 15°N. The scatter diagram is a useful tool to determine the degree of freeness of the flow. The area of scatter is a measure of the advection of q and hence the magnitude of $J(\psi, q)$. In free flow, $q = q(\psi)$, $J(\psi, q) = 0$ and the scatter collapses to a curve. However, in practice the functional



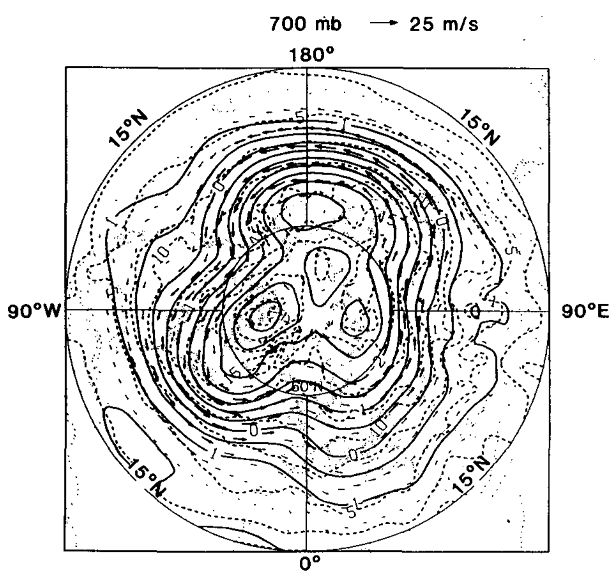


FIG. 2. The quasi-geostrophic potential vorticity (dotted lines) and streamfunction (continuous lines) for the nondivergent flow on the (a) 300 mb, (b) 500 mb and (c) 700 mb isobaric surfaces. Wind arrows are superimposed. Seasonal mean quantities are plotted—the December–February 1986/87 time mean. The unit of q is 10^{-5} s^{-1} and the unit for ψ is 10^7 m^2 s^{-1}. Details of the method of computation are given in Butchart (1989).

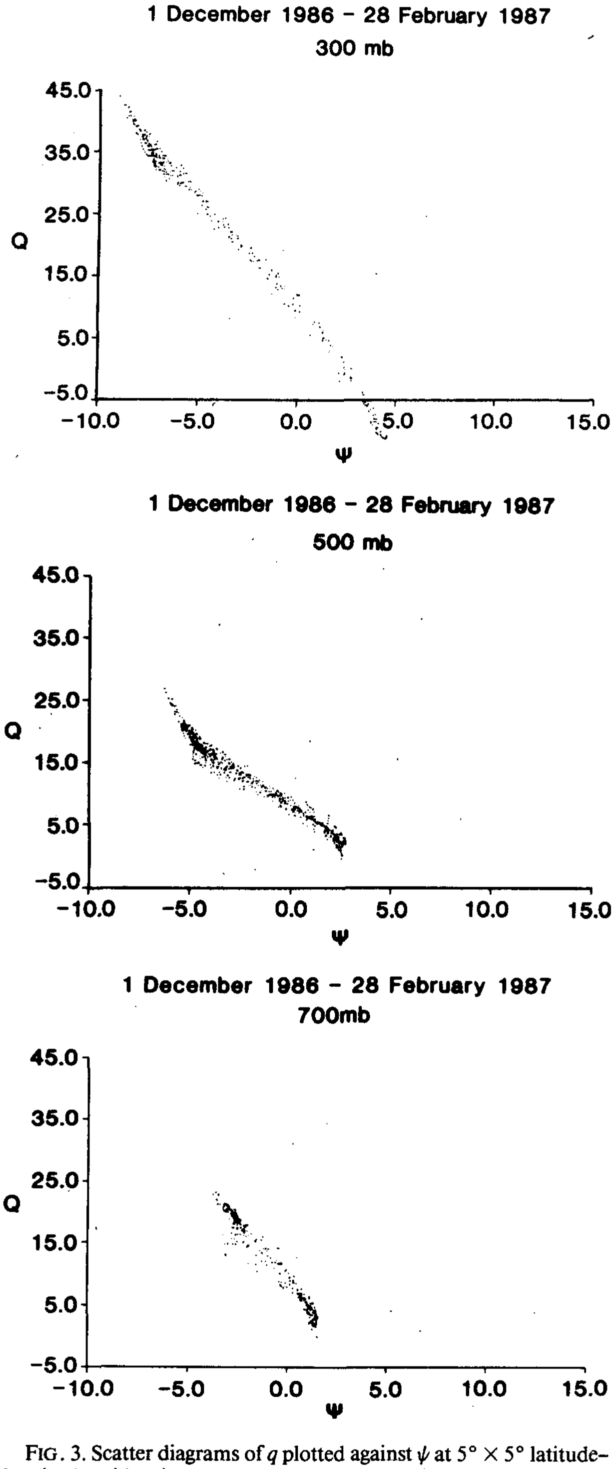


FIG. 3. Scatter diagrams of q plotted against ψ at $5° \times 5°$ latitude-longitude grid points over the Northern Hemisphere north of 15° for the December, January and February time-mean fields presented in Fig. 2: (a) 300 mb, (b) 500 mb, and (c) 700 mb. The unit for q is $10^{-5}\ s^{-1}$ and the unit for ψ is $10^7\ m^2\ s^{-1}$.

relationship between q and ψ is blurred because the flow departs from free mode form. The degree of departure can be measured by the quantity, I, which is the ratio of the scatter area to the area of the smallest rectangle with sides parallel to the q and ψ axes circumscribing the scatter (see Read et al. 1986). This quantity, I, can also be interpreted as the area average of the angle between ψ and q contours. It is evident that in a perfect free mode, the angle and I vanish. In Fig. 3, the degree of scatter is small allowing a functional relationship between q and ψ to be defined; the implied angles between the q and ψ contours are approximately 3, 7 and 15 degrees for the 300, 500 and 700 mb surfaces. It should be emphasized that in Fig. 2, q and ψ depart markedly from latitude circles and so the smallness of the scatter in Fig. 3 cannot be attributed to the dominance of the zonal flow.

A further remarkable fact is that the implied (q, ψ) curve is close to being linear. The reasons why the flow should favour linear (q, ψ) relationships are not clear but the empirical evidence presented in Fig. 3 provide a strong motivation for our interest in the class of finite amplitude stationary Rossby waves. It should be emphasized, however, that other classes of free-mode solution can be found (numerically) in the case when Λ changes smoothly along an isobaric surface—see for example Branstator and Opsteegh (1989). Indeed close inspection of Fig. 3 reveals departures from linearity particularly at low latitudes. However, the existence of further classes of free-mode solution will make the possibility of equilibration *more* rather than less likely.

4. Numerical illustrations

A three-level hemispheric spectral model at truncation T15 (described in the Appendix) is employed to thermally excite planetary waves. Diabatic heating of the form $Q = -\gamma(T - T^*)$ is adopted where T^* is a superposition of the stationary, free, finite amplitude Rossby waves studied in section 3. To simplify our discussion and present the equilibration mechanism in its purest form, all baroclinically unstable modes are suppressed by damping unforced wave modes with $m > 5$ on a time scale of ½ day. We shall have a particular interest in the excitation of planetary waves which have an equivalent barotropic vertical structure (no sign changes in the vertical).

Numerical experiments show that free modes can be readily excited and maintained (both grown from zero amplitude on an axisymmetric flow or maintained close to an initial free configuration) but only if a constraint is applied on the barotropic mode (or equivalently the surface winds). We have chosen to relax the (extrapolated) surface wind to zero by introducing a surface Ekman layer $\epsilon\nabla^2\psi_s$ where ψ_s is the surface pressure perturbation and ϵ is an Ekman spindown time. The vertical structure of the planetary wave is chosen so that ψ_s is zero. This is the only flow consistent with a solution of the unforced, undamped equations of motion; not only must $T = T^*$, but also $\psi_s = 0$ to ensure that frictionally driven Ekman layers at the surface are inactive. Thus as the planetary wave equilibrates toward T^* switching off the internal diabatic sources, the surface winds also spin down reducing to zero the mechanically driven vorticity sources at the surface.

If a constraint on the barotropic mode is not applied then it is not possible to equilibrate toward or maintain equivalent barotropic free-mode states because the Newtonian thermal forcing can only constrain the differences between the response at the three levels, not the absolute values.

Before going on to present results from our numerical experiments, it should be mentioned that the solution constructed in section 3 is not a perfect free mode of our numerical model and must be adjusted in two ways. First, the solid-body rotation represented by $\bar{\psi} = \sin\theta = \bar{P}_1{}^0(\sin\theta)$ cannot be exactly represented in the model since the zonal flow is required to be zero at the equator. Thus our hemispheric model does not carry a $\bar{P}_1{}^0$ component (see Appendix). Instead, $\bar{P}_1{}^0$ is approximated by a linear combination of $\bar{P}_2{}^0$ and $\bar{P}_4{}^0$ giving a jet at about 30°N, not unlike the observed wind pattern. This zonal configuration is, in fact, more realistic than a solid-body rotation which has a westerly maximum at the equator. It does not, however, have a band of equatorial easterlies; but see section 4c. Second, the vertical structure of the zonal flow, being different from that of the wavy components, does not extrapolate to zero at the surface. This problem is overcome by adjusting the zonal wind at level 1 to ensure that the extrapolated surface wind is indeed zero. This amounts to reducing the pole to equator temperature gradient at the lower thermodynamic level. With these two modifications, the quasi-free mode has a zonal jet at 30°N with speeds of 3.8, 11.4 and 24.1 m s^{-1} at the three levels and pole to equator temperature differences of 31.4 K and 40.9 K at the lower and upper thermodynamic levels respectively. The T^* field of the quasi-free mode is shown in Fig. 1b and is hardly distinguishable from the T of the free mode (not shown here) corresponding to Fig. 1a.

a. Equilibration at resonance

The model is initialized with an axisymmetric zonal flow and is thermally forced towards the quasi-free mode. The relaxation timescale for the wavy components of T^* is 7 days with an Ekman spindown time of 3.75 days. A numerical viscosity is applied to the vorticity on a timescale of 8 days for the shortest waves in the model (see Appendix). At day 40, the model has settled to a quasi-equilibrated state approaching the quasi-free mode, but with somewhat reduced amplitudes. Figure 4a shows T at 344 mb, Fig. 4b (T^*-T) at 344 mb and Fig. 4c the wavy component of T. The essential characteristics of the free mode of Fig. 1 are in evidence: domination by wavenumber 3 and 5 at high and middle latitudes respectively, the pattern of ridges and troughs, nearly vertical phase lines, a weak surface pressure pattern (the maximum amplitude is 3 mb), and low diabatic heating rates (on average less than 0.5 K day^{-1}). The maximum phase-shift of T relative to T^* is only 12 degrees of longitude. Scatter plots, Fig. 4d show an excellent functional relationship between q and ψ at levels 2 and 3 but with some bending of the curves; at level 1 there is somewhat more scatter but the flow is weak here, and the potential vorticity gradients small resulting in small tendencies. It is noteworthy that the thermal equilibration mechanism can operate successfully even though the zonal flow departs markedly from solid body form as the equator is approached.

b. The forced, off-resonant response

The nonequilibrated response can be illustrated by strengthening the zonal wind beyond that which can support the stationary, free Rossby wave whilst keeping the wavy components of T^* unchanged ($n = 8$). Accordingly, the strengthened zonal flow consists of $\bar{P}_2{}^0$ and $\bar{P}_4{}^0$ with a jet at 30°N of strength 6, 18, 32 m s^{-1} at levels 1 to 3, 2.2, 6.6 and 7.9 m s^{-1} stronger, respectively, than those of the quasi-free mode. This zonal flow is in fact appropriate for the free solution described in section 3 with $n = 7$ and vertical structure (1, 3, 5). The steady response is shown in Figs. 5a and 5b. The strong zonal flow has indeed blown the waves downstream: troughs in T now appear downstream of troughs in T^* rather than in phase with them. Furthermore, the wave amplitude has markedly decreased and the heating rate increased relative to the equilibrated response. In fact the flow now has all the hallmarks of a thermally forced response with an enhanced surface pressure pattern, and the planetary waves tilting westward with height. Recalling that $n = 7$ is the resonant point for this zonal flow, the linear model of section 2 suggests that the wave response corresponding to a T^* with $n = 8$ would be downstream and tilting westward, in accord with the numerical result. The response on the long-wavelength side of the resonant point, found in section 2 with the lower level ψ upstream of T^* and phase lines tilting eastward, can also be reproduced by the numerical model when the n of T^* is set to be less than 8 and the zonal flow of section 4a is adopted (i.e., the resonant point is $n = 8$). In this case the zonal wind is too weak to allow equilibration.

It is worth noting in passing that T^*-T (Fig. 5b) which could also be thought of as an anomaly field, might be mistaken for a wave train propagating from the tropics. As shown by Grose and Hoskins (1979) and Hoskins and Karoly (1981), the two-dimensional dispersion of Rossby waves away from a localized source have the form of wave-trains propagating along great circles (for a solid-body zonal flow). Here, of course, this characteristic pattern is not associated in

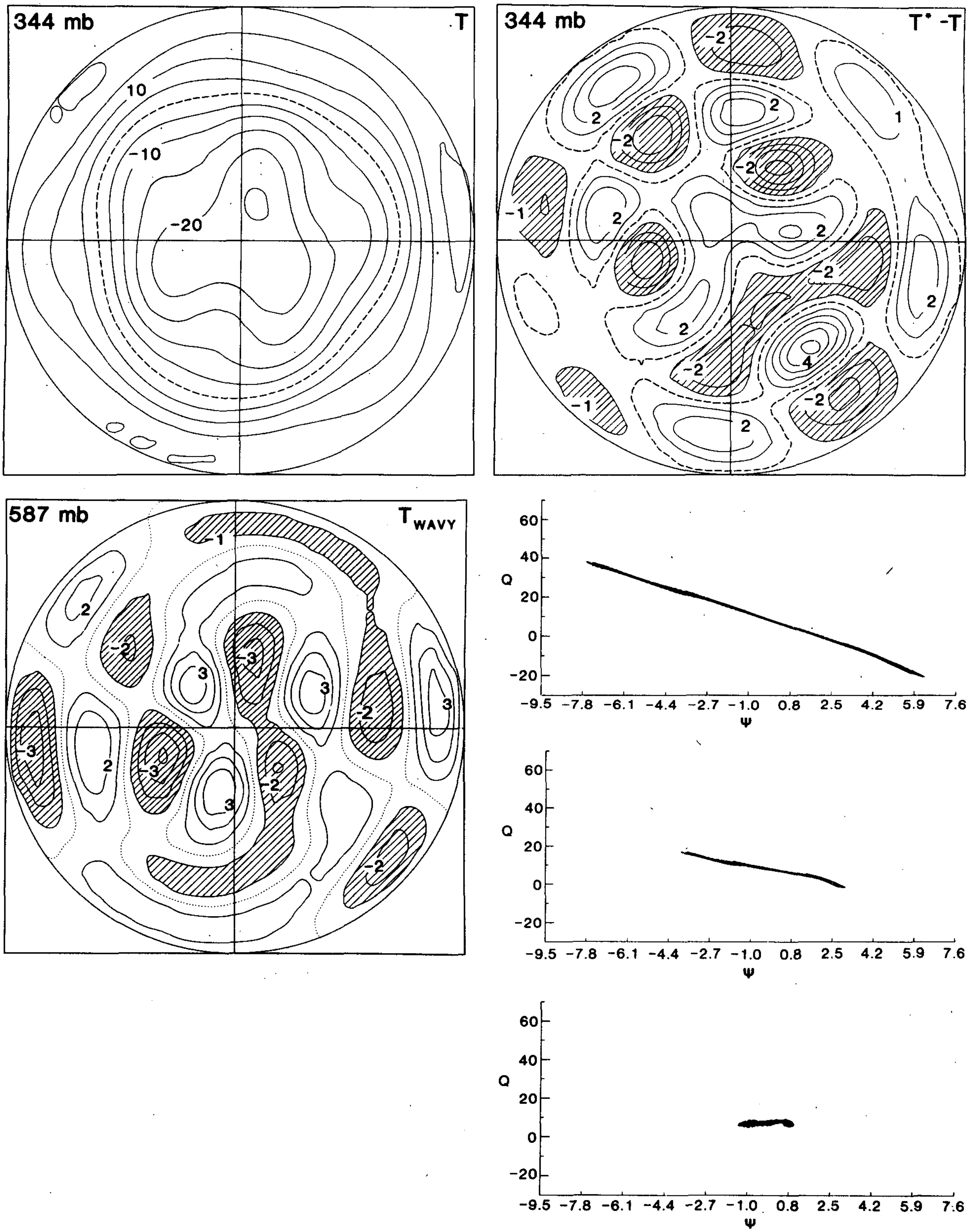
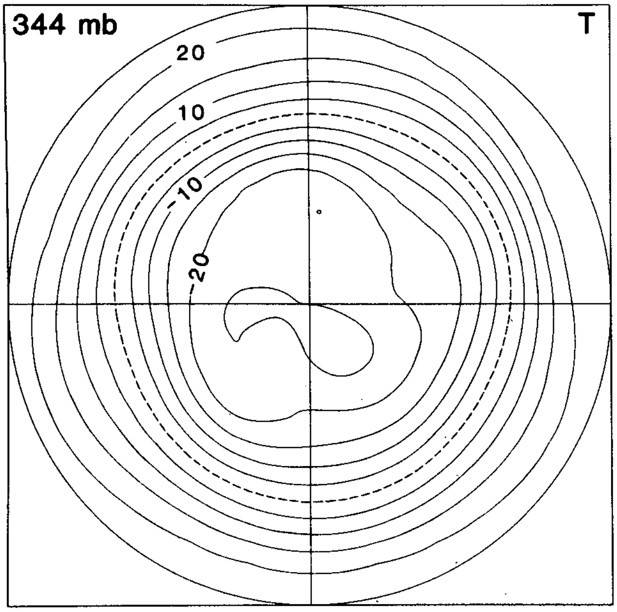


FIG. 4. (a) T at 344 mb near equilibration on day 40. Contour interval 5 K; (b) $T^{*} - T$ at 344 mb near equilibration on day 40. Contour interval 1 K; (c) wavy component of T at 587 mb; (d) scatter diagrams of q against ψ at the three levels of the model. Units same as Fig. 3.



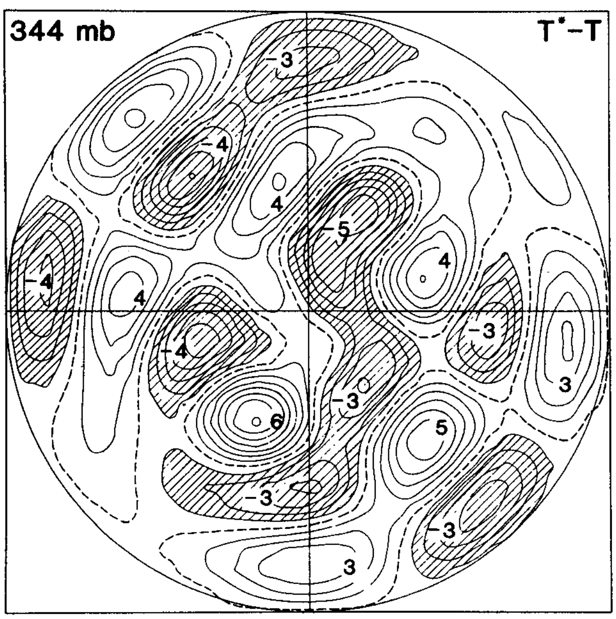


FIG. 5. As in Fig. 4a and 4b but for the off-resonant response on day 150.

any way in our model with wave propagation but is merely the difference between the forced and equilibrated responses. It is significant, however, that our mechanism can result in anomaly patterns which are reminiscent of Rossby wave trains.

c. Equilibration in the presence of equatorial easterlies

The relevance of the foregoing calculations to the atmosphere depends on whether or not stationary quasi-free waves can persist in realistic zonal flows. In fact, as reviewed, for example by Held (1983) and Webster (1983), the existence of finite amplitude stationary free-modes depends on significant reflection of Rossby waves from tropical easterlies. Any source of Rossby waves in middle latitudes will, according to linear theory, produce rays which are eventually attracted toward the zero-wind line separating equatorial easterlies and middle latitude westerlies. Linear theory suggests (Charney 1969) an effective barrier between middle latitudes and the tropics; modes will either be reflected or absorbed at some latitude poleward of the equator. If they are efficiently absorbed then large amplitude-free waves cannot persist in middle latitudes.

The literature on the role of critical lines is extensive but equivocal as to their reflective/absorptive properties [see for example Haynes and McIntyre (1987) and Killworth and McIntyre (1985)]. The observational evidence for significant absorption of wave activity in the tropics is largely based on diagnostics of the divergence of the Eliassen–Palm flux (see for example Edmon et al. 1980) but the steady free waves, which are the focus of our attention here, cannot have a signature in such statistics since they are not associated with any potential vorticity fluxes.

In view of the foregoing uncertainties we have taken a direct approach and attempted to demonstrate, or otherwise, that resonant excitation of free-waves can occur in middle latitudes in the presence of critical lines. Accordingly the equilibration experiments were repeated in the presence of a belt of tropical easterlies. Adjustments both to the zonal flow and T^* are made as follows. For the free mode shown in Fig. 1, with $n = 8$, wave amplitudes fall to zero near 24°N. The zonal flow south of this latitude is modified so as to create a broad band of easterlies which reach magnitudes of 6, 4.5 and 2.2 m s^{-1} at the equator at the lower, middle, and upper levels respectively of the model. The numerical model is modified to carry the $P_1{}^0$ component allowing nonzero zonal flow at the equator. The resulting winds happened to be configured in such a way that, as is strikingly evident in the observations—see, for example Fig. 4 of Sardeshmukh and Hoskins (1985)—the potential vorticity gradient in the equatorial band is very close to zero at the upper levels of the model. The easterly band is sufficiently broad that the waves cannot "tunnel through" to the equator. Care is taken to ensure that the wind field in middle latitudes is kept close to the solid body rotation form of Fig. 1. In addition to these adjustments of the zonal flow, the wavy components of T^* are chosen so that south of 24°N the wave amplitudes superpose to zero and remain unchanged north of this latitude. These modifications ensure that T^* and the zonal flow poleward of 24°N are indistinguishable from that of Fig. 1, but now a broad band of easterlies exist with almost uniform potential vorticity. Figure 6a shows the wavy compo-

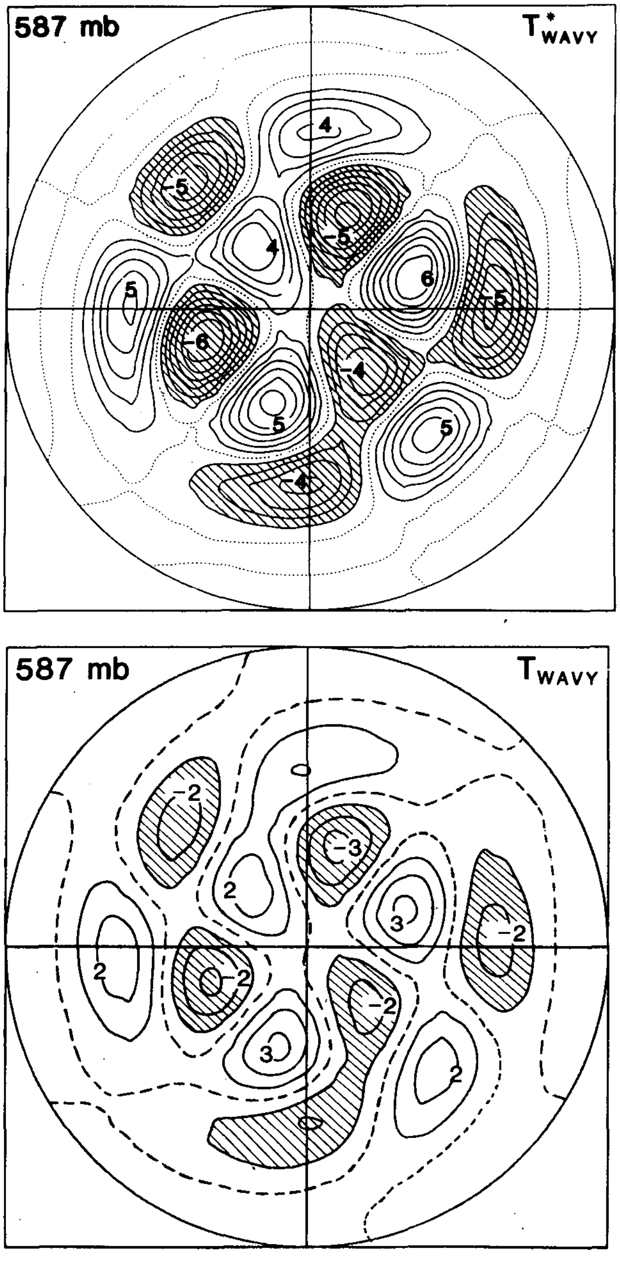


FIG. 6. (a) Wavy component of T^* at 587 mb and (b) wavy component of T at 587 mb in the presence of tropical easterlies.

nent of T^* and should be compared with Fig. 1c. Because T^* is zonal south of 24°N, any waves existing in the tropics will be thermally damped on a 7-day time scale. Furthermore, the enstrophy damping time scale in the model (see Appendix) was set so that the waves of the smallest meridional scale ($n = 15$) were damped on a ½ day time scale. This was thought to be sufficient to dissipate any equatorially propagating wave packets which, according to linear theory, are made up of waves which have increasingly rich meridional structure as the critical line is approached.

Figure 6b shows the thermal response at level one of the model and should be compared to Fig. 4c. We see that the amplitude of T in middle latitudes has diminished somewhat in the easterly wind case, but not to any significant extent. Evidently dissipation in the tropical easterlies of the model is not efficient enough to significantly modify the middle-latitude response. Furthermore, in the belt of easterlies, wave amplitudes are very small and there is no evidence of propagation to and reflection from the equatorial wall.

It appears then that equilibration can readily occur in the presence of a critical line embedded in a region where large-scale gradients of potential vorticity are anomalously weak. Consistent with all critical layer theory (for example, see McIntyre 1982), in our numerical experiment a belt of easterlies of uniform potential vorticity acts much like an equatorial wall reflecting significant fractions of wave energy incident from middle latitudes. Inspection of observed potential vorticity fields at upper levels in the tropics does indeed indicate that there are vast areas in which it is homogenized. However, localized regions of tropical westerlies do also exist, corresponding to enhanced meridional potential vorticity gradients which presumably act to "duct" wave energy equatorward. Nevertheless, given that planetary waves can amplify on the relatively short radiative–convective time scale, it seems very likely that in appropriate circumstances resonant excitation of free waves can take place provided that the ducting of energy through the tropical westerlies is not too large.

d. Oscillation of wave-number three

The numerical experiments described above illustrate the two extremes in the spectrum of thermal response. It can be envisaged that the thermally forced component of the planetary wave pattern will exhibit both responses, but the degree to which either limit is achieved will depend on the extent to which T^* projects onto stationary, free modes. This spectrum of response is well illustrated in the following experiment in which wavenumber three is thermally excited and subsequently proceeds to spontaneously oscillate between equilibrated and forced responses, the period of the oscillation being set by the radiative–convective relaxation time scale. The zonal flow appropriate to the free solution in section 3 for $n = 6$ and vertical structure (1, 3, 5) is strengthened by 3 m s^{-1} at each model level to give a jet at 30°N of strength 8, 24, and 42 m s^{-1}. The model is initialized with T set equal to a T^* given by $\bar{P}_6^{\,3}$ (shown in Fig. 7) corresponding to $n = 6$. The parameters of the model are as before except that now the thermal relaxation timescale is 10 days. After some initial adjustment wavenumber 3 begins to oscillate between forced and equilibrated states. By day 50 it is close to equilibration; from Figs. 8a–c it can be seen that the surface pressure pattern is featureless, the phase lines vertical, T almost in phase with T^* and the dia-

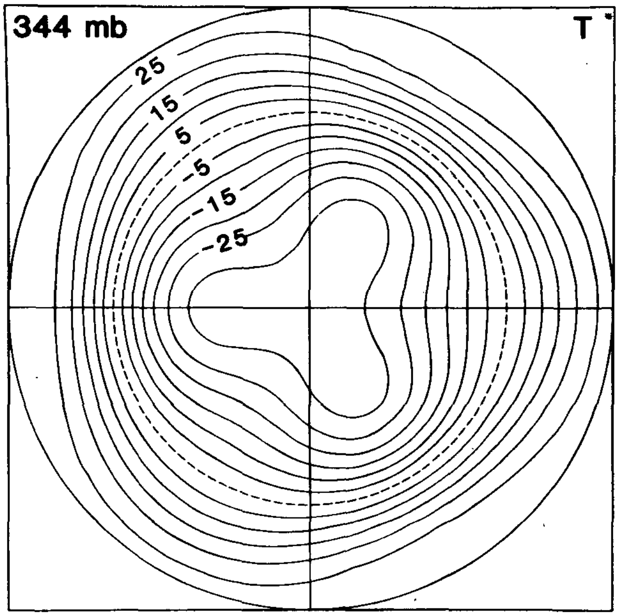


FIG. 7. T^* of the quasi-free mode at 344 mb for $n = 6$. Contour interval 5 K.

batic heating rates very low (in general less than 0.2 K d^{-1}). However, 10 days later, Figs. 8d–f, the wave has evolved into a more familiar state: T is now downstream of T^*, there is a strong surface pressure pattern, large phase shifts in the vertical (nearly 180° at high latitudes) and heating rates more than twice that at day 50. Wavenumber three continues to evolve until by day 75 it is near to equilibration again. Evidently a steady state cannot be achieved because the T^* towards which the waves are relaxing does not project sufficiently onto the free modes of the model.

It appears, then, that a thermal equilibration mechanism is capable of producing internal variability on radiative–convective time scales: the radiative "spring" pulls T back toward T^* on γ^{-1} time scales only for it to be blown downstream again by the strong westerlies.

5. Discussion

Diagnostic studies using scatter diagram techniques carried out specifically with free-mode ideas in mind (see for example Illari and Marshall 1984; Read, Rhines and White 1986; Butchart, Haines and Marshall 1989; and section 3 of the present study) show that there is a pronounced tendency for potential vorticity contours and streamlines to run parallel to one another. Such studies are revealing that much of the flow away from boundary layers is close to finite-amplitude free-mode behavior. A mechanism has been proposed, thermal equilibration, that may be an important process relaxing the atmosphere toward free-model states. This possibility has been investigated in a three-level hemispheric model driven by a Newtonian heating in an extension to a sphere of the β-plane channel studies of Mitchell and Derome (1983): attention has been focused on the class of finite amplitude Rossby waves. If a T^* is chosen to be compatible with a superposition of finite amplitude Rossby waves which are stationary on the hemisphere, then resonance and equilibration toward T^* is readily achieved provided a constraint on the surface wind is applied; i.e., that it too equilibrates to zero. A large amplitude wave can be maintained close to T^* with small diabatic heating rates even in the presence of equatorial easterlies. The more familiar thermal response can be illustrated by, say, increasing the zonal wind or suitably changing the form of T^* so that it is no longer compatible with a free solution. In this case a thermal trough appears downstream of the region of maximum cooling with the planetary waves tilting westward (or simply changing sign in the vertical), strong surface pressure patterns, and large diabatic heating rates. This is also illustrated as a class of downstream small-amplitude response in a simple linear model in section 2.

Our numerical experiments show that the planetary wave response can range from the thermally equilibrated to the forced regime on radiative–convective relaxation time scales suggesting that equilibration may be an important mechanism modulating low-frequency variability in the atmosphere. It should be emphasized, however, that in the present study global Rossby waves have been chosen as our free modes only for convenience to illustrate the essence of the idea. It might be expected that the global equilibration considered here might occur rather infrequently in the atmosphere. We suspect that local equilibration may be very much more common and relevant to, for example, atmospheric blocking. Indeed longitudinally confined Rossby wave-free modes can be grown on the sphere by exciting the appropriate spherical harmonics. Alternatively modon-like solutions could be candidate free-modes appropriate in the blocking context (see, for example, Pierrehumbert and Malguzzi 1984; Haines and Marshall 1987; Butchart et al. 1989).

Branstator and Opsteegh (1989) have stressed the importance of the density and distribution of free-mode states in phase space and consider free states in which q and ψ are nonlinearly related (obtained by a numerical variational approach). They find that there is a concentration of free-mode states in that part of phase-space in which the atmosphere resides and that generally they are tightly clustered. This has important implications for the present study for it appears that there may be a multiplicity of free-mode states available to the atmosphere, many more than contained in the class of stationary free Rossby waves alone. If this is the case then it is likely that T^* will project strongly onto this enlarged class of free-mode states and so resonant growth of flow patterns close to free-mode form might then be a ubiquitous feature of the atmosphere.

Our study would also seem to have some relevance

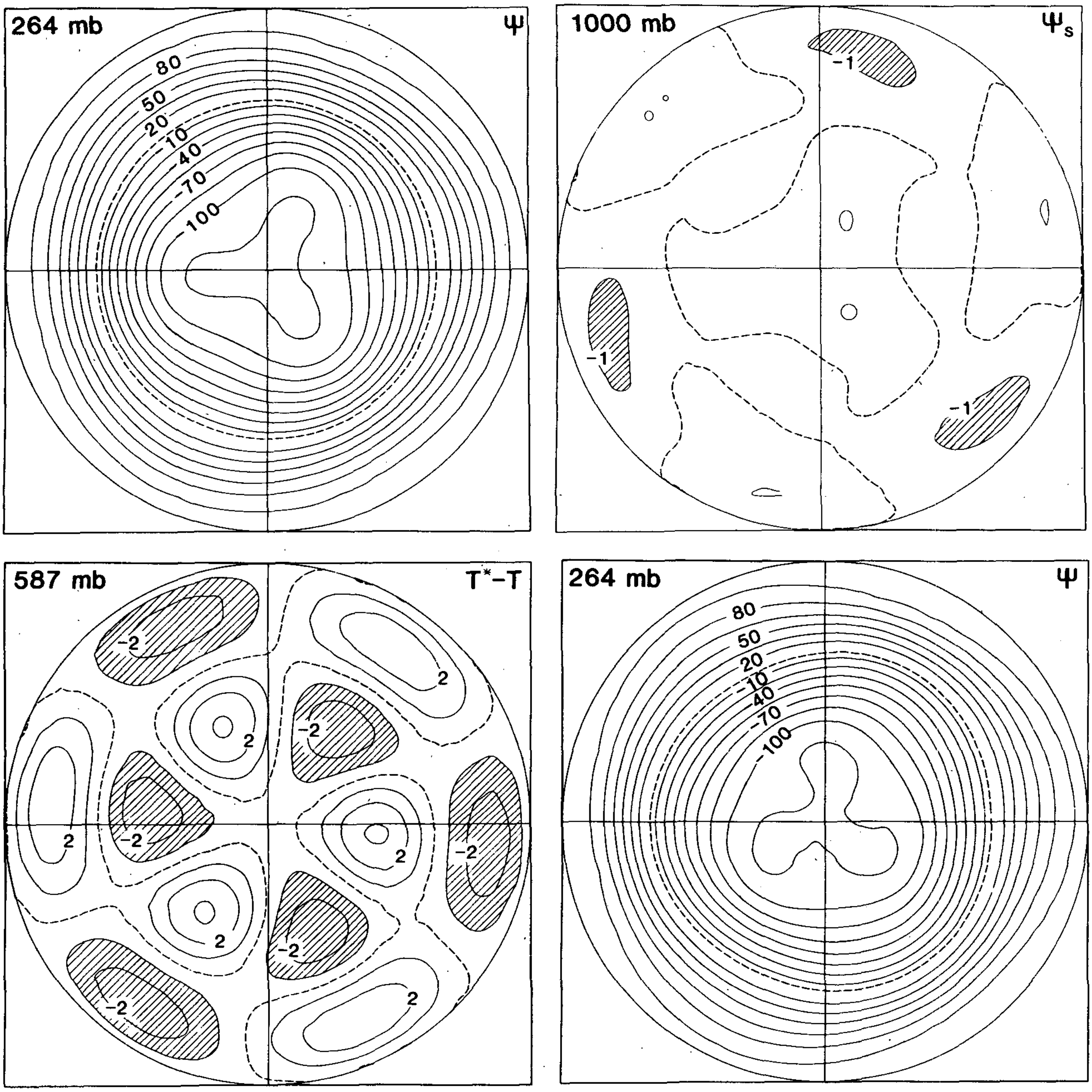
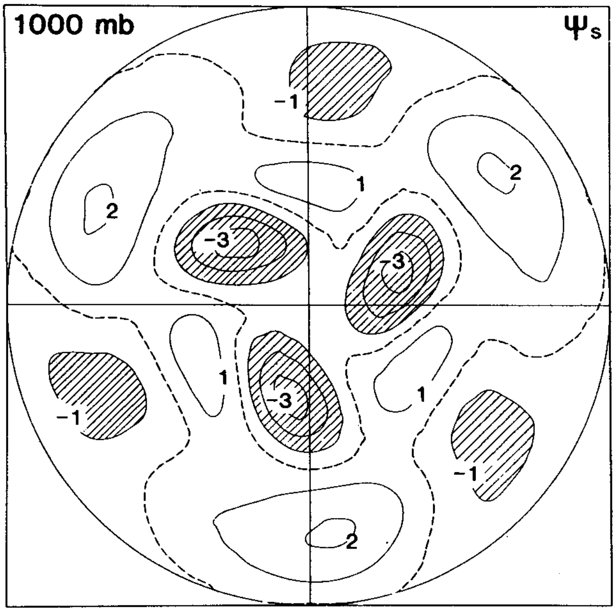


FIG. 8. The equilibrated (day 50, a, b and c) and off-resonant (day 60, d, e and f) responses of wavenumber 3 with $n = 6$. (a) and (d): Streamfunction at 264 mb. Contour interval 15 decameters; (b) and (e): Surface pressure pattern. Contour interval 1 mb; (c) and (f): T^*-T at 587 mb. Contour interval 1 K.

to the underlying dynamics of weather regimes. It is tempting to speculate that the anomalous weather regimes, ATL(±) and PAC(±), identified by Dole (1986), may be opposite extremes of a thermal response typified by thermally equilibrated and forced solutions, respectively. For example the PAC(+) pattern (equivalent to the negative phase of the PNA) is characterized by a large amplitude ridge over the eastern Pacific, the trough over the east coast of North America is upstream of its normal position and the Aleutian low is markedly weakened. These are all hallmarks of an equilibrated response. We also note that systematic errors in medium- and long-range forecasts are a strong function of flow regime (for example, see Arpe and Klinker 1986) and seem to be largest when the planetary wave amplitude is a maximum corresponding to ridging over the oceans. Since equilibration toward free mode form relies on the simultaneous "switching off" of the boundary layer physics and internal diabatic heating, great demands on the physical



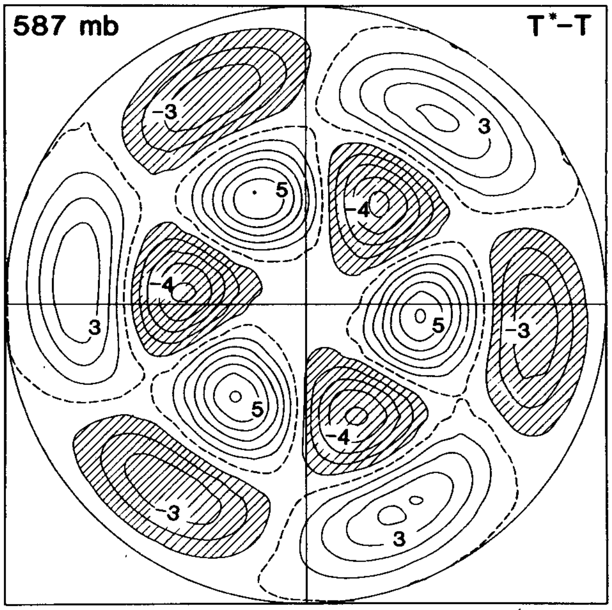


FIG. 8. (*Continued*)

parameterizations are made and so errors in the "physics" are likely to compromise the ability of the model to sustain these states. Conversely, the more zonal off-resonant response, with strong surface winds and fluxes and large diabatic heating rates may not be so sensitive to such errors.

Finally it is fascinating to observe that the difference fields between equilibrated and forced responses computed from our model are very reminiscent of wave trains propagating along great circles (for example, see Fig. 5b). Indeed the anomaly patterns over the Pacific documented by Dole (1986) are the two phases of the ubiquitous "PNA" pattern. Could it be that the PNA pattern is in part a consequence of the planetary waves locally switching between these two thermal states? Furthermore, errors in the forecast model tend to have the appearance of wave trains and are often attributed to Rossby wave propagation triggered by errors in the tropical heating field. The present study provides an additional mechanism; the anomaly fields between equilibrated regimes and the climatology have a similar characteristic signature.

Acknowledgements. We should like to thank the Gassiot Grants Committee of the Meteorological Office whose support made this study possible. We acknowledge numerous conversations with Drs. Glenn Shutts and Andy White of the Meteorological Office and Keith Haines of Imperial College.

APPENDIX

The Quasi-geostrophic Hemispheric Model

Standard quasi-geostrophic equations expressed in spherical polar geometry are used for a dry atmosphere with $h = \ln(p_0/p)$ as a height coordinate where p_0 is the constant, average surface-pressure. A conventional level discretization is used with, in our case, two thermodynamic levels midway between three kinematic levels, as shown in Fig. 9.

$$\frac{\partial T_0}{\partial h} + \kappa T_0 = 29.3 \text{ K}$$

has been used in the definition of the static stability, S, at both thermodynamic levels. The Jacobian of ψ and q in the potential vorticity equation, Eq. (1), is written in spherical coordinates thus

$$J(\psi, q) = \frac{\partial(Vq)}{\partial\mu} + \frac{1}{1-\mu^2}\frac{\partial(Uq)}{\partial\lambda}$$

where $V = v\cos\theta$, $U = u\cos\theta$, $\mu = \sin(\text{lat})$ and λ is the longitude.

The horizontal streamfunction fields at each level of the model are expanded into spherical harmonics of odd parity for the wave fields and even parity for the zonal-mean fields:

$$\bar{\psi} = \sum_{n=2, n \text{ even}}^{14} \overline{\psi_n}(h, t)\bar{P}_n^0(\mu)$$

$$\psi' = \sum_{\substack{m=1\\ n-m \text{ odd}}}^{14} \sum_{n=m+1}^{15} [\psi_{cn}^m(h, t)\cos m\lambda + \psi_{sn}^m(h, t)\sin m\lambda]\bar{P}_n^m(\mu)$$

where $\bar{P}_n^m$ are the normalized associated Legendre functions. This mixed parity formulation is required in this hemispheric model for the following reason. To

$\omega = 0$ at 202 mb

ψ_3 at 264 mb

T_2 at 344 mb

ψ_2 at 450 mb

T_1 at 587 mb

ψ_1 at 766 mb

ψ_s at 1000 mb

FIG. 9. Vertical discretization of the spectral model.

ensure zero net cross-equatorial flow, we have chosen to set $\bar{u} = 0$ at the equator (this can be seen to be sufficient by considering the zonally averaged u-momentum equation at the equator where the Coriolis torque does not vanish in a quasi-geostrophic model). If $\bar{u}$ is to vanish at the equator, then $\bar{\psi}$ must be expanded in terms of even Legendre polynomials.

Evaluation of divergence terms is accomplished by the grid-transform technique. A scale-selective viscosity,

$$\frac{\mu}{[15(15+1)]^3}\nabla^6(\nabla^2\psi)$$

where μ^{-1} is the damping time scale in days, is applied to the vorticity equation to dissipate small-scale enstrophy. The damping timescale is normalized with respect to waves of the shortest meridional scale in the model, $n = 15$ for the experiments described here.

The model was built in the Department of Physics, Imperial College, by Drs. Mansbridge and Shutts. For further details see Shutts (1983).

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