Journal of Marine Research, 69, 647–657, 2011

A pedagogical example of Eulerian and Lagrangian flows induced by Rossby wave rectification in an ocean basin

by John Marshall[1]

ABSTRACT

The time-mean flow of eddying wind-driven gyres is used as a tutorial example of the relationship between Eulerian and Lagrangian mean flow. The Eulerian mean in the far field of the baroclinically unstable jet is shown to be well represented as the rectified flow of Rossby basin modes. The Stokes drift of particles released in the wave field all but cancel out the Eulerian mean, resulting in vanishingly small Lagrangian mean flow. By reformulating the interaction between eddies and mean flow in terms of, so-called, residual mean velocities and residual eddy fluxes, it becomes clear that only the component of the eddy potential vorticity flux that crosses mean potential vorticity gradients, must be parameterized.

1. Introduction

Here we study the wave rectification due to the nonlinear interaction of barotropic basin normal modes. Aspects of this problem have been previously studied by Pedlosky (1965) and Harrison and Robinson (1979). Here we show that the theory is relevant to the mean flow generated by the barotropic far field of an eddy resolving baroclinic numerical ocean model.

As a simple analytic description of the interior eddy field we use the "meander induced forcing" (MIF) model of Harrison and Robinson (1979), in which waves are generated by oscillating a boundary (the meandering Gulf Stream, for example). In the inviscid limit the "eddy-like" solutions to the MIF problem resemble basin normal modes and so the calculation of the rectified flow is similar to that of Pedlosky (1965) who, in a study of the time-dependent circulation, excited normal mode solutions at resonant frequency.

We present this problem as a pedagogical example of wave-mean flow interaction which forces us to think about and define Eulerian and Lagrangian mean flows and grapple with the nature of eddies and their driving of mean flow. This subject was of great interest to Melvin Stern throughout his career, beginning with his very first paper on the 'moving flame experiment' (Stern, 1959), stemming from his graduate work at MIT. There, as reviewed

1. Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge MA 02139. ***email: jmarsh@mit.edu***

by Whitehead (1975), a heater moving under a horizontal cylindrical annulus of liquid, generates mean flow.

What is of interest in the example explored here is that Eulerian mean flow is driven almost entirely by eddy potential vorticity (PV) fluxes which are directed *parallel* to mean PV contours. The residual-mean flow all but vanishes, however, because the Eulerian mean is balanced by the Stokes drift of particles circling in the wave field. We argue, and demonstrate by appropriately transforming the prognostic equations for mean PV, that it is only eddy PV fluxes directed across the mean PV contours — i.e. the 'residual flux' — that needs to be parameterized. Our discussion is also relevant to recent developments and applications of 'residual mean' theory to the ocean (e.g. Marshall and Radko, 2003; Plumb and Ferrari, 2005) and to the reformulation of ocean models in terms of residual-mean quantities (e.g. Ferreira and Marshall, 2006; Zhao and Vallis, 2008) — and in attempts to understand and parameterize ocean eddies in coarse resolutions models — see, for example, Holland and Rhines (1980), Rhines and Young (1982), Marshall (1984), Holm and Nadiga (2003).

We begin in Section 2 by studying the mean flow observed in eddying wind-driven, quasi-geostrophic gyres. In Section 3 we set out a theoretical framework in which we will attempt to understand what is going on. In Section 4 we present solutions of the theoretical model that describe, we believe, the eddying and mean flows observed in the numerical model. Finally, in Section 5 we place our results in the wider context of residual mean theory and draw out the lessons of our study for attempts to formulate and parameterize the eddy fluxes in coarse-resolution ocean models.

2. Eulerian mean flow of eddying gyres: a numerical example

We describe numerical simulations from a $2-$level single gyre configuration of the quasi-geostrophic model described in Marshall *et al.* (1988). Driven by an idealized sinusoidal wind-stress pattern (there is no thermal forcing), a gyral pattern is spun up in excess of the Sverdrup prediction through inertial recirculation. The flow is baroclinically unstable and eddies and waves extract energy from the mean flow and play a role in shaping its character. In Figure 1(a) we show plots of instantaneous streamfunction ψ and quasi-geostrophic potential vorticity q, in upper and lower levels of the model. Coexisting with the meandering, nonlinear jet in the northwest corner of the gyre, we observe a strong interior eddy field which has a marked barotropic component. Note how the q contours undulate in the interior of the gyre as Rossby waves, excited by the meandering jet, propagate westwards. The Eulerian, time-mean flow is shown in Figure 1(b). The upper level flow has the characteristic hallmark of a wind-driven gyre with significant recirculation, as discussed in the barotropic context by, for example, Veronis (1966). The mean flow in the lower level — and this is the focus of attention of the present article — comprises a matrix of closed, counter-rotating gyres which have a scale less than that of the ocean basin. These gyres are strong and persistent, a striking feature of the Eulerian flow. We shall see below that they are entirely eddy driven.

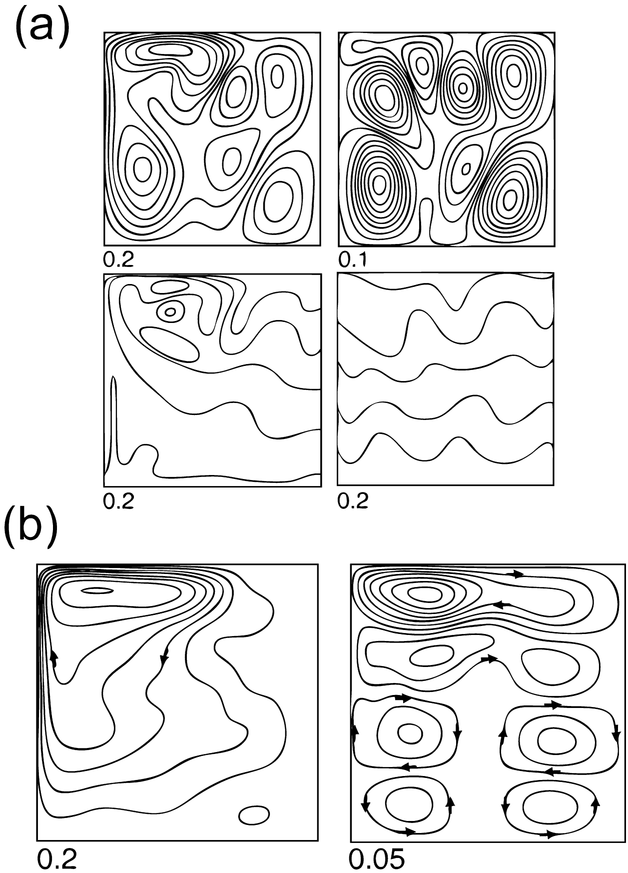

Figure 1. (a) Instantaneous streamfunction in the (left) upper layer and (right) lower layer, both normalized by the Sverdrup transport. (bottom) Instantaneous qgpv in the (left) upper layer and (right) lower layer, nondimensionalized wrt βL. (b) The time-mean (left) upper layer and (right) lower layer streamfunction, normalized by the Sverdrup transport. The contour interval is indicated.

In the remainder of this article we attempt to explain, in terms of eddy-mean flow interaction theory, how these gyres are set up by the eddy field. We shall find that they are a consequence of the rectification of Rossby basin modes which are resonantly excited by the meandering of the jet along the northern boundary of the model. However, we will also discover that these Eulerian-mean gyres, although real, give one a completely wrong impression about the trajectory of fluid parcels and Lagrangian-mean circulation. In fact, the Eulerian mean flow in the interior of the basin is almost entirely canceled out by an equal and opposite circulation associated with Stokes drift, so that the Lagrangian mean (or residual) flow is very close to zero. This, we will argue, has important lessons as we reformulate ocean models in terms of residual-mean quantities.

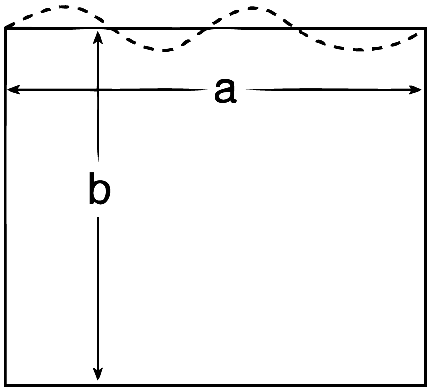

Figure 2. In the 'meander induced forcing' model, the spatial and temporal variation of the streamfunction is specified on the northern boundary of a basin of width a and meridional extent b.

3. Theoretical model of eddy-mean flow interaction

The ocean model is a barotropic fluid on a β-plane contained in a rectangular basin of dimension a by b, as sketched in Figure 2. We suppose that there are rigid walls to the east, west and south on which the streamfunction is constant. The basin is open to the north, however, where we specify the streamfunction as a function of x and t. Dissipation is by bottom friction.

The vorticity equation is (nondimensionalized)

$$\frac{\partial}{\partial t}\nabla^2\psi + RJ(\psi, \nabla^2\psi) + \frac{\partial\psi}{\partial x} = -\varepsilon\nabla^2\psi \tag{1}$$

where $R = U_S/\beta L^2$ (where U_S is a Sverdrup velocity scale, β the beta effect and L the scale of the basin) is a Rossby number for the vorticity equation and ε is the bottom friction coefficient. In the solution shown in Figure 1, $R = 2 \times 10^{-5}$ — see the appendix of Marshall *et al.* (1988) for a discussion of the nondimensionalization and typical parameters.

Supposing, then, that R is small we expand the streamfunction in powers of R thus:

$$\psi = \psi^{(0)} + R\psi^{(1)} + \ldots\ldots$$

The zeroth order problem is then linear and given by

$$\frac{\partial}{\partial t}\nabla^2\psi^{(0)} + \frac{\partial\psi^{(0)}}{\partial x} = -\varepsilon\nabla^2\psi^{(0)}. \tag{2}$$

The first-order correction (the rectified flow) is given by

$$\frac{\partial}{\partial t}\nabla^2\psi^{(1)} + \frac{\partial\psi^{(1)}}{\partial x} = -\varepsilon\nabla^2\psi^{(1)} - J(\psi^{(0)}, \nabla^2\psi^{(0)}). \tag{3}$$

We will solve Eq. (2) for the eddies using the MIF model of Harrison and Robinson (1979), compute the eddy vorticity flux divergence forcing term in Eq. (3), and then solve Eq. (3) for the rectified flow. First, though, we discuss general properties of the $\psi^{(1)}$ flow which do not depend on particular solutions to (2).

Although $\overline{\psi^{(0)}} = 0$ (where $\overline{\langle - \rangle}$ at a point represents a time-average long compared to an eddy period), nonlinear interactions can generate a nonzero steady flow, $\overline{\psi^{(1)}} \neq 0$. Taking the time-average of (3) it becomes

$$\frac{\partial}{\partial x}\overline{\psi^{(1)}} = -\varepsilon\nabla^2\overline{\psi^{(1)}} - \overline{J(\psi^{(0)}, \nabla^2\psi^{(0)})}. \tag{4}$$

Provided that the eddy forcing term is of large scale, and the frictional coefficient sufficiently small, we can expect an interior Sverdrup balance, but here forced by eddy fluxes associated with the zeroth order flow.

The enstrophy equation for the zeroth order flow is:

$$\varepsilon\overline{(\nabla^2\psi^{(0)})^2} + \overline{v^{(0)}\nabla^2\psi^{(0)}} = 0, \tag{5}$$

which says that in the absence of dissipation the meridional flux of potential vorticity is everywhere zero. Although the streamfunction at the open northern boundary is varying with time, even here there can be no time-averaged transport of vorticity in the absence of dissipation. A consideration of the energetics shows that there is also no work done at the boundary if $\varepsilon = 0$.

In the inviscid limit Eq. (4) can be integrated to yield a particularly simple form:

$$\overline{\psi^{(1)}} = -\overline{u^{(0)}\nabla^2\psi^{(0)}}, \tag{6}$$

where $\overline{\psi^{(1)}}$ on the eastern boundary has been set to zero. Interestingly the solution (6) satisfies the boundary conditions at both meridional walls where there can be no normal vorticity flux. The mean flow is thus entirely driven by a component of the potential vorticity flux which is parallel to the planetary vorticity contours!

So, unlike the classical Sverdrup balance, our eddy-driven inviscid Sverdrup solution can satisfy both eastern and western boundary conditions without the need for frictional boundary layers. This is possible because the eddy forcing term in (4) does not generate vorticity (or enstrophy) unless it is being dissipated. In the presence of dissipation, Eq. (5) implies a southwards component of the potential vorticity flux to offset local dissipation of eddy enstrophy. Solutions to (4) in this case will have a boundary layer character. Let us now consider in more detail the nature of the solutions.

4. Solutions for the eddy and rectified flow

a. The eddy field

The work of Harrison and Robinson (1979) suggests that linear theory, as represented by Eq. (2), is relevant to interior mesoscale model eddies and that certain boundary forced solutions may be useful as a description of the ocean mesoscale variability. In their MIF model, solutions of (4) are driven by imposing an oscillatory northern boundary condition, a crude parameterization of the nonlinear processes occurring in the transient northern boundary current.

The solution of (2) satisfying $\psi_E^{(0)} = \psi_W^{(0)} = \psi_S^{(0)} = 0$ and

$$\psi_N^{(0)} = R_e\{e^{-i\sigma t} F(x)\} \text{ with } F(0) = F(a) = 0$$

is

$$\psi^{(0)} = R_e \left\{ e^{-i(\sigma t + kx)} \sum_{m=1}^{\infty} c_m \sin(m\pi x) \sin \lambda y \right\} \tag{7}$$

$$\text{where } k = \frac{1}{2(\sigma + i\varepsilon)};\ \lambda^2 = k^2 - m^2\pi^2$$

$$\text{and } c_m = 2 \int_0^1 \sin(m\pi x) F(x) e^{-ikx} \sin\left(\frac{\lambda b}{a}\right) dx.$$

Properties of the solution (7) for small ε are discussed at some length in HR. Solutions are classified as "trapped" or "eddy-like" depending on whether there are closed streamlines or not. The existence of the "eddy-like" solutions does not depend on the particular form of the forcing function $F(x)$. It is only required that the solution be sinusoidal in y with at least one interior node: i.e.

$$\lambda^2 > \frac{a^2\pi^2}{b^2}. \tag{8}$$

HR go on to compare their MIF solutions with eddy streamfunctions derived from various general circulation models and find in each case that single inviscid "eddy-like" term of (7)

$$\psi^{(0)} = c_m \cos\left(\sigma t + \frac{x}{2\sigma}\right) \sin(m\pi x) \sin \lambda y \tag{9}$$

satisfactorily describes the scale and pattern of evolution of the far field barotropic stream function if σ is chosen as the dominant eddy frequency of the model. $\psi^{(0)}$ is a basin mode (see Pedlosky, 1987), a wave travelling towards the west modulated by sines, as sketched in Figure 3.

b. The rectified flow

The time independent vorticity forcing, due to the nonzero correlations between $\hat{\mathbf{z}} \times \nabla\psi^{(0)}$ and $\nabla^2\psi^{(0)}$ in a single mode of the above form, can now be calculated, and then (6) gives the rectified flow.

The vorticity forcing is

$$\overline{v^{(0)}\nabla^2\psi^{(0)}} = 0;\ \overline{u^{(0)}\nabla^2\psi^{(0)}} \sim \sin^2(m\pi x)\sin(2\lambda y)$$

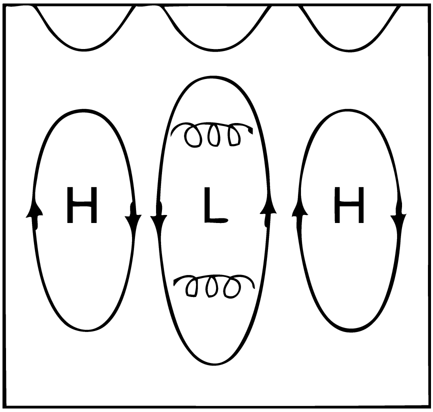

Figure 3. Sketch of the $\psi^{(0)}$ solution from the MIF model showing high (H) and low (L) circulation patterns. In the inviscid limit $\psi^{(0)}$ has the form of a Rossby basin mode. The trajectories of particles released in to the basin mode solution are indicated by the curly arrows.

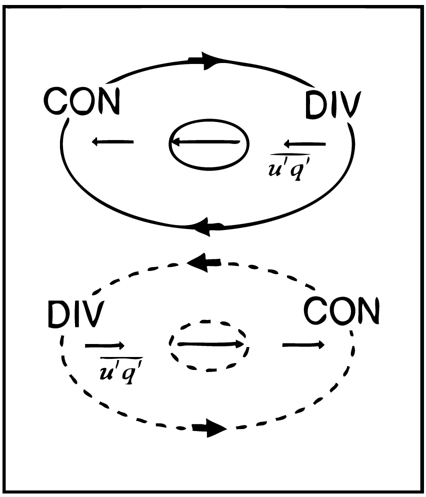

Figure 4. Sketch of the rectified flow, $\psi^{(1)}$, Eq. (10), and the driving eddy fluxes $\overline{\mathbf{u}'q'}$, for the gravest $\psi^{(0)}$ solution, Eq. (9) (with $m = 1$, $\lambda = \pi$). Regions of eddy flux divergence and convergence are indicated.

and so

$$\overline{\psi^{(1)}} = -\overline{u^{(0)}\nabla^2\psi^{(0)}} \sim -\sin^2(m\pi x)\sin(2\lambda y). \tag{10}$$

The rectified flow and the driving eddy fluxes for the gravest mode are shown in Figure 4.

There is a mean anticyclonic gyre to the north with westward pointing vorticity fluxes, and a cyclonic gyre to the south with eastward pointing fluxes. The mean flow is driven across the f contours by the eddy flux divergence.

The form of the rectified flow, Eq. (10), has been derived before. Pedlosky (1965) considered a barotropic model driven by a fluctuating wind-stress curl. At resonant forcing (the amplitude was limited by friction) the form of the linear response is that of a normal mode, and hence the rectified flow is given by (10).

There is a straightforward physical interpretation of the pattern of eddy fluxes shown above. If particles are released in the $\psi^{(0)}$ field given by Eq. (9), then they are seen to rotate anticyclonically and drift eastward to the north of the latitude of maximum wave amplitude and cyclonically and drift westward to the south, as sketched in Figure 3. It takes a particle one wave period to complete the circuit. A parcel rotating anticyclonically carries more relative vorticity from east to west than it returns from west to east (it conserves $\zeta + f$ and f varies with y). There is thus a net transfer of vorticity from east to west. Vorticity is not transferred meridionally — a parcel carries as much relative vorticity north as it brings back south. On the southern flank of the wave train particles released in to the wave rotate in the opposite direction, and so the sign of the vorticity transfer is reversed.

The amount of vorticity transferred will be greater the greater the meridional extent of the excursion. This depends on the local amplitude of the disturbance: the modulation in space of the travelling wave. Thus the east-west flux will be a divergent flux if the wave amplitudes vary zonally, as for a wave of the form Eq. (9). It is this divergence which drives the Eulerian mean flow $\psi^{(1)}$. In the example given here it is the envelope of the basin mode required to satisfy the boundary conditions on the meridional walls which modulate the wave train rather than dissipation.

The modulating envelope causes the particles gyrating in the $\psi^{(0)}$ field to systematically drift in one direction. Particles displaced northwards (say) at one longitude return southwards at another where the wave amplitude is different, and so they are not returned to their original latitude. This causes particles to migrate. The direction of migration depends on the sense of rotation of the parcels (see Fig. 3), and on whether the wave amplitude is increasing or decreasing. The rate at which the particles drift — the Stoke's drift velocity — is exactly balanced by the 1st order Eulerian mean flow. Thus the Lagrangian mean flow (mean velocity of a parcel over a wave period) is zero, as it must be because there is no dissipation and the mean potential vorticity contours intersect boundaries:

$$\mathbf{v}_L = \underbrace{R\overline{\mathbf{v}^{(1)}}}_{\overline{v}} + \underbrace{\overline{\boldsymbol{\eta}^0 \bullet \nabla \mathbf{v}^{(0)}}}_{\mathbf{v}^*} = 0 \tag{11}$$

where $\boldsymbol{\eta}^0$ is the displacement of the fluid parcel from a release point.

The relevance of the above picture the mean flow of our eddying gyre can be seen in Figure 5 where $\overline{\mathbf{v}'q'}$ and $\overline{\psi}$ are plotted for the numerical experiment shown in Figure 1. In the northern quarter of the domain we see a strong anticyclonic gyre associated with the jet flowing along, and recirculating to the south of, the northern boundary. Here the eddy fluxes have a significant component directed down the large-scale mean $\overline{q}$ gradient. However, in the remainder of the domain to the south, we observe eddy fluxes which have

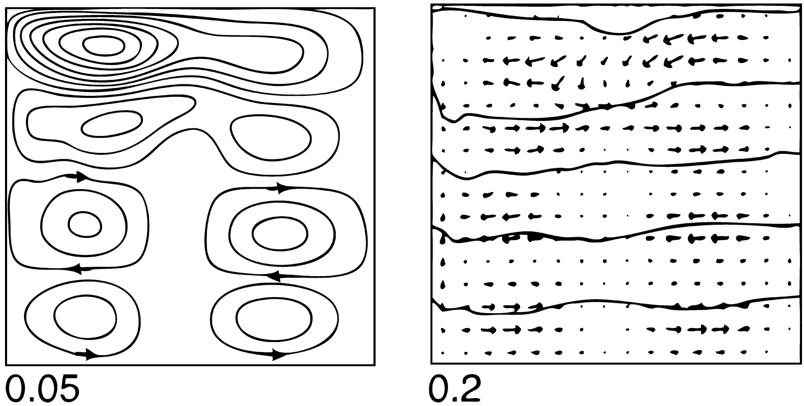

Figure 5. The time-mean lower layer streamfunction. (right) The eddy qgpv fluxes, $\overline{\mathbf{u}'q'}$. This $\left(m = 2, \lambda = \frac{\pi}{2}\right)$ pattern should be compared to the $(m = 1, \lambda = \pi)$ solution sketched in Figure 4. The contour interval is indicated.

a predominantly zonal component directed parallel to $\overline{q}$. We observe a 2×3 matrix of Eulerian-mean gyres, $\overline{\psi}$, driven by these fluxes.

5. Interpretation in terms of residual-mean theory

The equation governing the evolution of the mean potential vorticity $\overline{q}$ can be written:

$$\frac{\partial \overline{q}}{\partial t} + \mathbf{v} \cdot \nabla \overline{q} + \nabla \cdot \overline{\mathbf{v}'q'} = \mathcal{F} - \mathcal{D}. \tag{12}$$

Guided by residual-mean theory — see Andrews and McIntyre (1976) and Andrews *et al.* (1987) — we separate the eddy q flux in to a 'skew' component which is parallel to the $\overline{q}$ contours and a 'residual' component which has a component across the $\overline{q}$ contours. Specifically we choose to write:

$$\overline{\mathbf{v}'q'}_{skew} = (\overline{u'q'}, \overline{u'q'}s_q) \tag{13}$$

where $s_q = -\frac{\overline{q}_x}{\overline{q}_y}$ is the slope of the $\overline{q}$ contours and the remaining, residual component is:

$$\overline{\mathbf{v}'q'}_{res} = (0, \overline{v'q'} - \overline{u'q'}s_q). \tag{14}$$

Note that in the limit of the $\overline{q}$ contours being coinicident with latitude circles ($s_q = 0$), then

$$\overline{\mathbf{v}'q'}_{skew} = (\overline{u'q'}, 0)$$
$$\overline{\mathbf{v}'q'}_{res} = (0, \overline{v'q'}).$$

The divergence of the skew flux is associated entirely with an advective process because of the identity:

$$\nabla \cdot \left(\overline{\mathbf{v}'q'}_{skew}\right) = \mathbf{v}^* \cdot \nabla \overline{q} = J_{x,y}(\psi^*, \overline{q})$$

where

$$\psi^* = \frac{\overline{u'q'}}{\overline{q}_y}. \tag{15}$$

We call $\mathbf{v}^*$ the 'eddy-induced' velocity and ψ^* the eddy-induced streamfunction.

In the rectified basin mode problem worked out in Section 4, ψ^* exactly cancels out $\overline{\psi}$, Eq. (10): it induces a circulation of an equal and opposite sign to the Eulerian-mean gyres seen in Figure 5, resulting in a vanishingly small residual-mean flow.

In terms of residual-mean quantities, Eq. (12) can be written:

$$\frac{\partial \overline{q}}{\partial t} + \mathbf{v}_{res} \cdot \nabla \overline{q} = \mathcal{F}_{res} + \mathcal{F} - \mathcal{D} \tag{16}$$

where

$$\mathbf{v}_{res} = \overline{\mathbf{v}} + \mathbf{v}^* = \widehat{\mathbf{z}} \times \nabla(\overline{\psi} + \psi^*) \tag{17}$$

corresponds to Eq. (11) of our analytical model and

$$\mathcal{F}_{res} = -\frac{\partial}{\partial y}\overline{v'q'}_{res}. \tag{18}$$

Thus we see that in an eddying ocean it is $\mathbf{v}_{res}$ and not $\overline{\mathbf{v}}$ that advects $\overline{q}$ and it is $\overline{v'q'}_{res}$, not $\overline{\mathbf{v}'q'}$, that must be parameterized.

In the eddying model that has been the focus of our attention here, $\mathcal{F}_{res}, \mathcal{F}, \mathcal{D} \longrightarrow 0$ and so $\mathbf{v}_{res} \cdot \nabla \overline{q} \approx 0$ i.e. $\psi_{res} = \overline{\psi} + \psi^* \longrightarrow 0$. The matrix of Eulerian-mean gyres evident in Figure 5 does not imply material transport of properties and need not be parameterized.

6. Conclusions

We have explored the rectification of Rossby basin modes in which an Eulerian-mean flow, $\overline{\psi}$, is driven by an eddy pv flux directed entirely along $\overline{q}$ contours. The Stokes drift of particles released in the wave field exactly cancels the Eulerian mean and the resulting residual-mean flow vanishes. The problem is highly idealized and yet is successful in explaining the Eulerian-mean circulations observed in the interior of eddying, wind-driven gyres. It also provides an excellent pedagogical illustration of the differences between Eulerian and Lagrangian-mean flow, Stokes drift, residual flow and residual fluxes.

Important lessons of the study are:

1. the simplification and clarification of eddy-mean flow processes that result from a reformulation of the equations in terms of residual-mean velocities and residual eddy pv fluxes
2. the central role of the residual pv flux in driving residual-mean circulation
3. the focus of eddy parameterizations should be the component of the flux directed across time-mean potential vorticity contours.

Acknowledgments. JM acknowledges NSF's longstanding support of his work in the GFD of the atmosphere and ocean.

REFERENCES

Andrews, D. G., J. R. Holton, and C. B. Leovy. 1987. Middle Atmosphere Dynamics, volume 40 of International Geophysics Series. Academic Press, 489 pp.

Andrews, D. and M. McIntyre. 1976. Planetary waves in horizontal and vertical shear: the generalized Eliassen–Palm relation and the zonal-mean acceleration. J. Atmos. Sci., *33*, 2031–2048.

Ferreira, D. and J. Marshall. 2006. Formulation and implementation of a "residual-mean" ocean circulation model. Ocean Model, *13*, 86–107.

Harrison, D. E. and A. R. Robinson. 1979. Boundary-forced planetary waves: a simple model mid-ocean response to strong current variability. J. Phys. Oceanogr., *9*, 919–929.

Holland, W. R. and P. B. Rhines. 1980. An example of eddy-induced ocean circulation. J. Phys. Oceanogr., *10*, 1010–1031.

Holm, D. and B. Nadiga. 2003. Modeling mesoscale turbulence in the barotropic double-gyre circulation. J. Phys. Oceanogr., *33*, 2355–2365.

Marshall, J. 1984. Eddy mean flow interaction in a barotropic ocean model. Quart. J. Roy. Meteor. Soc., *110*, 573–590.

Marshall, J., A. J. G. Nurser, and R. Brugge. 1988. On the time-averaged flow of quasi-geostrophic wind-driven gyres. J. Geophys. Res., *93*, 15427–15436.

Marshall, J. and T. Radko. 2003. Residual mean solutions for the Antarctic Circumpolar Current and its associated overturning circulation. J. Phys. Oceanogr., *33*, 2341–2354.

Pedlosky, J. 1965. A study of time-dependent ocean circulation. J. Atmos. Sci., *22*, 267–272.

—— 1987. Geophysical Fluid Dynamics, 2nd ed., Springer, NY, 710 pp.

Plumb, R. A. and R. Ferrari. 2005. Transformed Eulerian-mean theory. Part 1: Nonquasigeostrophic theory for eddies on a zonal-mean flow. J. Phys. Oceanogr., *35*, 165–174.

Rhines, P. B. and W. R. Young. 1982. Homogenization of potential vorticity in planetary gyres. J. Fluid Mech., *122*, 347–367.

Stern, M. S. 1959. The moving flame experiment. Tellus, *11*, 175–179.

Veronis, G. 1966. Wind-driven ocean circulation. Part 2: Numerical solutions of the non-linear problem. Deep-Sea Res., *13*, 31–55.

Whitehead, J. A. 1975. Mean flow generated by circulation of a β-plane: an analogy with the moving flame experiment. Tellus, *27*, 358–364.

Zhao, R. R. and G. Vallis. 2008. Parameterizing mesoscale eddies with residual and Eulerian schemes, and a comparison with eddy-permitting models. Ocean Model, *23*, 1–12.

Received: *17 May, 2011*; revised: *4 December, 2011*.